

# PRACTISE EXERCISES FOR TEST 3 - MATH 103DW

3.6 (Implicit Differentiation)

5.5 (Differentiation Involving Logarithms)

4.1 (Applications of 1st Derivative)

4.2 (Applications of 2nd Derivative)

4.3 (Curve Sketching) BONUS

(1) Find  $dy/dx$

a-  $2x^2 - y^2 = 3$

b-  $xy^2 + y = 5$

c-  $\sin(xy) = 1$

d-  $e^{x+y} = 3 + x$

e-  $x^2 \ln y = y$

f-  $\sqrt{x} - 2xy = 16$

g-  $x^3y - xy = 2$

h-  $y \cos(x + y) = x$

i-  $\ln(x^2y^2) = \frac{x^2}{2}$

j-  $x^2e^y + x = y$

(2) Find the equation of the tangent line to the curve at the given point.

a-  $\ln(xy) + x = y^2$  at the point  $(1, 1)$

b-  $x^3y^2 - 3xy = -2$  at the point  $(1, 2)$

c-  $f(x) = 4 - x^2 - \ln(\frac{1}{2}x + 1)$  at the point  $(0, 4)$

d-  $f(x) = x^3 \ln x$  at the point  $(1, 0)$

e-  $y^2e^{xy} = 2y + x$  at the point  $(0, 1)$

(3) Find  $dy^2/d^2x$  (your answer should be in terms of  $x$  and  $y$  only)

a-  $x^2 + y^2 = 25$

b-  $y^2 = x^3$

c-  $1 - xy = x - y$

d-  $y^2 = 4x$

(4) Find the derivative (your answer should be in terms of x only)

a-  $y = \ln(x^2 + 1)$

b-  $f(x) = \ln(\ln x)$

c-  $y = \ln(\cos^2 x)$

d-  $y = \ln \frac{x(x-1)^{\frac{3}{2}}}{\sqrt{x+1}}$

e-  $y = \ln\left(\frac{x}{x^2+1}\right)$

f-  $y = x^2 \ln(x^2)$

g-  $f(x) = \sin(2x) \ln(x^2)$

h-  $y = (x+1)^x$

i-  $y = x^{\cos x}$

j-  $y = (\ln x)^x$

k-  $y = \ln(x^x)$

(5) Find the points on the curve of the function where the tangent line is horizontal.

a-  $f(x) = -\frac{1}{8}(x+2)^2(x-4)^2$

b-  $f(x) = \frac{x}{x^2+1}$

(6) Find all relative extrema of the function.

a-  $f(x) = \frac{x+3}{x^2}$

b-  $f(x) = 2x^3 + 3x^2 - 12x$

c-  $f(x) = x\sqrt{16-x^2}$

d-  $f(x) = \frac{x^5-5x}{5}$

(7) Find the relative extrema, the intervals where the function is increasing/decreasing, the inflection points, the intervals where the function is concave up/down. (Bonus, sketch the graph indicating intercepts as well)

a-  $f(x) = x^2(6 - x)^2$

b-  $f(x) = x^3 - 9x^2 + 27$

c-  $f(x) = x^3(x - 4)$

d-  $f(x) = x^3 - 6x^2 + 12x$

e-  $f(x) = 2x^4 - 8x + 3$

f-  $f(x) = (x - 1)^{\frac{1}{3}}$

g-  $f(x) = x^{\frac{2}{3}} - 4$

h-  $f(x) = \frac{1}{4}x^4 - 2x^2$

i-  $f(x) = x^4 - 4x^3 + 2$

j-  $f(x) = -(x - 5)^2$