Dawson College: **Calculus II**: 201-NYB-05-S02: Fall 2009

Name: _____
Student ID: _____

Test 1

This test is graded out of 47 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Formula:

$\sum_{i=1}^{n} c = cn$ where c is a constant $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ $\quad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

Question 1. *(4 marks)* Integrate the following indefinite integral:

$$\int 2\frac{1}{\sqrt[3]{x}} + 3\csc x - \frac{3}{\sqrt{3-x^2}}\,dx$$

Question 2. *(5 marks)* Evaluate the definite integral using first principles *(i.e. limit process)*:

$$\int_0^2 -x^2 + 2x - 1\,dx$$

Question 3. *(3 marks)* Integrate the following indefinite integral:

$$\int \frac{e^{\ln y^2}}{y}\, dy$$

Question 4. *(5 marks)* Integrate the following indefinite integral:

$$\int \frac{z^3 - z^2 + z - 1}{z^2 + 5}\, dz$$

Question 5. *(5 marks)* Evaluate the following definite integral:

$$\int_1^2 (x^2 + x)(2x^3 + 3x^2)^2 \, dx$$

Question 6. *(5 marks)* Differentiate the following expression:

$$\frac{d}{dx}\left[\int_x^{\arctan x} f(t) \, dt\right]$$

Question 7. *(4 marks)* Find the average of the function $f(x) = \tan x$ on the interval $[-\frac{\pi}{6}, \frac{\pi}{6}]$.

Question 8. Given that $f(-x) = -f(x)$, $g(-x) = g(x)$, $\int_0^a f(x)\,dx = 3$ and $\int_0^a g(x)\,dx = 5$, evaluate the following definite integrals:

a. *(2 marks)*

$$\int_a^a 2g(x)\,dx$$

b. *(2 marks)*

$$\int_{-a}^a 3f(x)\,dx$$

c. *(2 marks)*

$$\int_{-a}^a -2g(x)\,dx$$

Question 9. *(5 marks)* Evaluate the following definite integral:

$$\int_{\frac{\pi}{4}}^{0} \frac{\sec^2 x}{1+\tan^2 x}\, dx$$

Question 10. *(5 marks)* Integrate the following indefinite integral:

$$\int \frac{\sec(\arcsin 2x)}{\sqrt{1-4x^2}}\, dx$$

Bonus Question.

a. *(1 mark)* State the First Fundamental Theorem of Calculus.

b. *(1 mark)* State the Mean Value Theorem.

c. *(3 marks)* Prove the First Fundamental Theorem of Calculus.