Name: Y. Lamontagne
Student ID: _____

Assignment 2

This assignment is graded out of 10 marks. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(1 mark)* Write the equation of the plane $-3x+2y-7z=21$ in parametric form.

let $x=0, y=0 \Rightarrow -3(0)+2(0)-7z=21 \Rightarrow z=-3$ $\therefore P_0(0,0,-3)$

let $x=0, z=0 \Rightarrow -3(0)+2y-7(0)=21 \Rightarrow y=\frac{21}{2}$ $\therefore P_1(0,\frac{21}{2},0)$

let $y=0, z=0 \Rightarrow -3x+2(0)-7(0)=21 \Rightarrow x=-7$ $\therefore P_2(-7,0,0)$

$\vec{P_0P_1} = P_1-P_0 = (0,\frac{21}{2},0)-(0,0,-3) = (0,\frac{21}{2},3)$

$\vec{P_0P_2} = P_2-P_0 = (-7,0,0)-(0,0,-3) = (-7,0,3)$

$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix} + s\begin{pmatrix} 0 \\ \frac{21}{2} \\ 3 \end{pmatrix} + t\begin{pmatrix} -7 \\ 0 \\ 3 \end{pmatrix}$

Question 2. *(1 mark)* Write the equation of the plane $(x,y,z) = (2,-1,4)+s(3,1,3)+t(-2,-1,2)$ where $s, t \in \mathbb{R}$ in general form.

$\vec{n} = \vec{u} \times \vec{v} = \begin{matrix} 3 & -2 \\ 1 & -1 \\ 3 & 2 \end{matrix} = \left(\begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix}, -\begin{vmatrix} 3 & -2 \\ 3 & 2 \end{vmatrix}, \begin{vmatrix} 3 & -2 \\ 1 & -1 \end{vmatrix}\right) = (5, -12, -1)$

$\therefore 5x-12y-z=d$ sub $(2,-1,4)$ into eqn to solve for d.

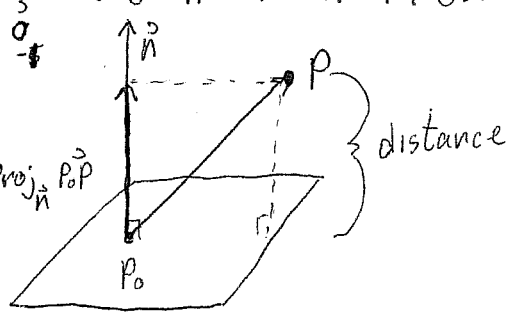
$5(2)-12(-1)-4=d$

$18=d$

$\therefore 5x-12y-z=18$

Question 3. *(3 marks)* Find the distance between the point $P(3,4,5)$ and the plane $(x,y,z) = (1,0,3)+s(2,-1,0)+t(3,0,-1)$ where $s, t \in \mathbb{R}$.

$\vec{n} = \vec{u} \times \vec{v} = \begin{matrix} 2 & 3 \\ -1 & 0 \\ 0 & -1 \end{matrix} = \left(\begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix}, -\begin{vmatrix} 2 & 3 \\ 0 & -1 \end{vmatrix}, \begin{vmatrix} 2 & 3 \\ -1 & 0 \end{vmatrix}\right) = (1, 2, 3)$



$\therefore x+2y+3z=d$

sub $(1,0,3)$ into above to solve in d

$(1)+2(0)+3(3)=d$

$4=d$

$\therefore x+2y+3z=4$

and $\vec{n}=(1,2,3)$

$\vec{P_0P} = P-P_0 = (3,4,5)-(1,0,3) = (2,4,2)$

$\text{proj}_{\vec{n}} \vec{P_0P} = \frac{\vec{n} \cdot \vec{P_0P}}{\vec{n} \cdot \vec{n}} \vec{n}$

$= \frac{(1,2,3)\cdot(2,4,2)}{(1,2,3)\cdot(1,2,3)}(1,2,3)$

$= \frac{16}{14}(1,2,3)$

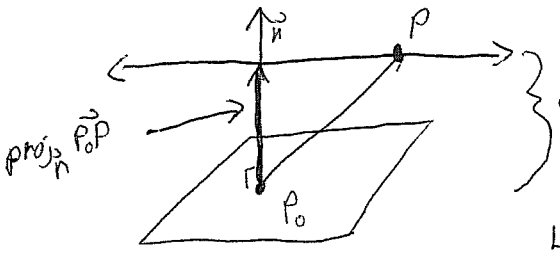
$= \left(\frac{16}{14}, \frac{32}{14}, \frac{48}{14}\right)$

$\|\text{proj}_{\vec{n}} \vec{P_0P}\| = \sqrt{\left(\frac{16}{14}\right)^2+\left(\frac{32}{14}\right)^2+\left(\frac{48}{14}\right)^2}$

$= \sqrt{\frac{256}{14}}$

$= \frac{16}{\sqrt{14}}$

Question 4. *(2 marks)* Find the distance between the line $(x,y,z) = (2+t, 2, 5+3t)$ and the plane $3x-4y-z=20$.



the normal is $\vec{n} = (3,-4,-1)$

a point on the line is when $t=0$

$P(2,2,5)$

Lets find a point on the plane, let $x=0, z=0$

$3(0)-4y-0=20$

$y=-5$ $\therefore P_0(0,-5,0)$

and $\vec{P_0P} = P - P_0 = (2,2,5) - (0,-5,0) = (2,7,5)$

so $\text{proj}_{\vec{n}} \vec{P_0P} = \dfrac{\vec{n}\cdot\vec{P_0P}}{\vec{n}\cdot\vec{n}}\vec{n} = \dfrac{(3,-4,-1)\cdot(2,7,5)}{(3,-4,-1)\cdot(3,-4,-1)}(3,-4,-1)$

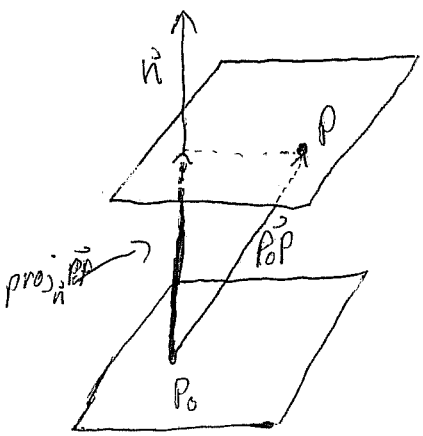
$= \dfrac{-27}{26}(3,-4,-1)$

$= \left(\dfrac{-81}{26}, \dfrac{-108}{26}, \dfrac{27}{26}\right)$

$\therefore \| \text{proj}_{\vec{n}} \vec{P_0P} \| = \sqrt{\left(\frac{-81}{26}\right)^2 + \left(\frac{-108}{26}\right)^2 + \left(\frac{27}{26}\right)^2}$

$= \sqrt{\dfrac{729}{26}}$

Question 5. *(3 marks)* Find the distance between the two parallel planes: $2x - y + z = 1$ ① and $-4x+2y-2z=-1$ ②.



Lets find a point P on plane ①, let $x=0, y=0$

$2(0)-0+z=1$

$z=1$ $\therefore P(0,0,1)$

Lets find a point P_0 on plane ②, let $x=0, z=0$

$-4(0)+2y-z(0)=-1$

$y=-\frac{1}{2}$ $\therefore P_0(0,-\frac{1}{2},0)$

so $\vec{P_0P} = P - P_0 = (0,0,1) - (0,-\frac{1}{2},0)$

$= (0,\frac{1}{2},1)$

and we know $\vec{n} = (2,-1,1)$

$\therefore \text{proj}_{\vec{n}} \vec{P_0P} = \dfrac{\vec{P_0P}\cdot\vec{n}}{\vec{n}\cdot\vec{n}}\vec{n} = \dfrac{(0,\frac{1}{2},1)(2,-1,1)}{(2,-1,1)\cdot(2,-1,1)}(2,-1,1)$

$= \dfrac{\frac{1}{2}}{6}(2,-1,1) = \left(\dfrac{2}{12}, \dfrac{-1}{12}, \dfrac{1}{12}\right)$

and $\| \text{proj}_{\vec{n}} \vec{P_0P} \| = \sqrt{\left(\frac{2}{12}\right)^2 + \left(\frac{-1}{12}\right)^2 + \left(\frac{1}{12}\right)^2} = \sqrt{\dfrac{6}{12^2}} = \dfrac{\sqrt{6}}{12}$