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Assignment 3

This assignment is graded out of 10 marks. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* Maximize $p = 8x_1 + 9x_2 + 4x_3$ subject to the constraints

$$\begin{aligned} x_1 + x_2 + 2x_3 &\le 2 \\ 2x_1 + 3x_2 + 4x_3 &\le 3 \\ 7x_1 + 6x_2 + 2x_3 &\le 8 \end{aligned}$$

into $\Rightarrow$ eqn.

$$\begin{aligned} x_1 + x_2 + 2x_3 + s_1 &= 2 \\ 2x_1 + 3x_2 + 4x_3 + s_2 &= 3 \\ 7x_1 + 6x_2 + 2x_3 + s_3 &= 8 \\ -8x_1 - 9x_2 - 4x_3 + p &= 0 \end{aligned}$$

Simplex matrix:

$$\left[\begin{array}{cccc|cccc} 1 & 1 & 2 & 1 & 0 & 0 & 0 & 2 \\ 2 & 3 & 4 & 0 & 1 & 0 & 0 & 3 \\ 7 & 6 & 2 & 0 & 0 & 1 & 0 & 8 \\ -8 & -9 & -4 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$r = 2/1 = 2$
$r = 3/3 = 1$
$r = 8/6 = 1.\overline{3}$

pivot row ↖ pivot column

$\frac{1}{3}R_2$

$$\left[\begin{array}{cccc|cccc} 1 & 1 & 2 & 1 & 0 & 0 & 0 & 2 \\ 2/3 & 1 & 4/3 & 0 & 1/3 & 0 & 0 & 1 \\ 7 & 6 & 2 & 0 & 0 & 1 & 0 & 8 \\ -8 & -9 & -4 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$-R_2 + R_1 \to R_1$
$-R_2 + R_3 \to R_3$
$-9R_2 + R_4 \to R_4$

$$\left[\begin{array}{cccc|cccc} 1/3 & 0 & 2/3 & 1 & -1/3 & 0 & 0 & 1 \\ 2/3 & 1 & 4/3 & 0 & 1/3 & 0 & 0 & 1 \\ 3 & 0 & -6 & 0 & -2 & 1 & 0 & 2 \\ -2 & 0 & 8 & 0 & 3 & 0 & 1 & 9 \end{array}\right]$$

$r = 1/1/3 = 3$
$r = 1/2/3 = 3/2$
$r = 2/3$ ← pivot row

↑ pivot column

$\frac{1}{3}R_3$

$$\left[\begin{array}{cccc|cccc} 1/3 & 0 & 2/3 & 1 & -1/3 & 0 & 0 & 1 \\ 2/3 & 1 & 4/3 & 0 & 1/3 & 0 & 0 & 1 \\ 1 & 0 & -2 & 0 & -2/3 & 1/3 & 0 & 2/3 \\ -2 & 0 & 8 & 0 & 3 & 0 & 1 & 9 \end{array}\right]$$

$-\frac{1}{3}R_3 + R_1 \to R_1$
$-\frac{2}{3}R_3 + R_2 \to R_2$
$2R_3 + R_4 \to R_4$

$$\left[\begin{array}{cccc|cccc} 0 & 0 & 4/3 & 1 & -1/9 & -1/9 & 0 & 7/9 \\ 0 & 1 & 8/3 & 0 & 7/9 & -2/9 & 0 & 5/9 \\ 1 & 0 & -2 & 0 & -2/3 & 1/3 & 0 & 2/3 \\ 0 & 0 & 4 & 0 & 5/3 & 2/3 & 1 & 31/3 \end{array}\right]$$

max is $31/3$
at $x_1 = 2/3$, $x_2 = 5/9$, $x_3 = 0$

Question 2. *(5 marks)* Maximize $p = x_1 + 2x_2 + 4x_3 + 5x_4$ subject to the constraints

$x_1 + x_2 + x_4 \le 44$ into
$2x_1 + x_2 + 2x_3 + 5x_4 \le 200$ $\Rightarrow$
$x_1 + x_3 \le 50$ eqn

$$\begin{aligned} x_1 + x_2 \quad\quad + x_4 + s_1 \quad\quad\quad\quad &= 44 \\ 2x_1 + x_2 + 2x_3 + 5x_4 \quad + s_2 \quad\quad &= 200 \\ x_1 \quad\quad + x_3 \quad\quad\quad + s_3 \quad &= 50 \\ -x_1 - 2x_2 - 4x_3 - 5x_4 \quad\quad + p \quad &= 0 \end{aligned}$$

$$\left[\begin{array}{cccc|cccc|c} 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 44 \\ 2 & 1 & 2 & 5 & 0 & 1 & 0 & 0 & 200 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 50 \\ -1 & -2 & -4 & -5 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$r = 44/1 = 44$
$r = 200/5 = 40 \leftarrow$ pivot row

↑ pivot column

$$\frac{1}{5}R_2 \left[\begin{array}{cccc|cccc|c} 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 44 \\ 2/5 & 1/5 & 2/5 & 1 & 0 & 1/5 & 0 & 0 & 40 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 50 \\ -1 & -2 & -4 & -5 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$\begin{array}{l} -R_2 + R_1 \to R_1 \\ \\ \\ 5R_2 + R_4 \to R_4 \end{array} \left[\begin{array}{cccc|cccc|c} 3/5 & 4/5 & -2/5 & 0 & 1 & -1/5 & 0 & 0 & 4 \\ 2/5 & 1/5 & 2/5 & 1 & 0 & 1/5 & 0 & 0 & 40 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 50 \\ 1 & -1 & -2 & 0 & 0 & 1 & 0 & 1 & 200 \end{array}\right]$$

$r = 40/2/5 = 100$
$r = 50/1 = 50 \leftarrow$ pivot row

↑ pivot column

$$\begin{array}{l} \frac{2}{5}R_3 + R_1 \to R_1 \\ -\frac{2}{5}R_3 + R_2 \to R_2 \\ \\ 2R_3 + R_4 \to R_4 \end{array} \left[\begin{array}{cccc|cccc|c} 1 & 4/5 & 0 & 0 & 1 & -1/5 & 2/5 & 0 & 24 \\ 0 & 1/5 & 0 & 1 & 0 & 1/5 & -1/5 & 0 & 20 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 50 \\ 3 & -1 & 0 & 0 & 0 & 1 & 2 & 1 & 300 \end{array}\right]$$

$r = 24/4/5 = 30 \leftarrow$ pivot row
$r = 20/1/5 = 100$

↑ pivot column

$$\frac{5}{4}R_1 \left[\begin{array}{cccc|cccc|c} 5/4 & 1 & 0 & 0 & 5/4 & -1/4 & 1/2 & 0 & 30 \\ 0 & 1/5 & 0 & 1 & 0 & 1/5 & -1/5 & 0 & 20 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 50 \\ 3 & -1 & 0 & 0 & 0 & 1 & 2 & 1 & 300 \end{array}\right]$$

$$\begin{array}{l} -1/5R_1 + R_2 \to R_2 \\ \\ \\ R_1 + R_4 \to R_4 \end{array} \left[\begin{array}{cccc|cccc|c} 5/4 & 1 & 0 & 0 & 5/4 & -1/4 & 1/2 & 0 & 30 \\ -1/4 & 0 & 0 & 1 & -1/4 & 1/4 & -1/2 & 0 & 14 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 50 \\ 17/4 & 0 & 0 & 0 & 5/4 & 3/4 & 5/2 & 1 & 330 \end{array}\right]$$

∴ max is 330 attained at
$x_1 = 0, x_2 = 30$
$x_3 = 50, x_4 = 14$