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Assignment 4

This assignment is graded out of 10 marks. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* Minimize $C = 2x_1 + x_2 + 3x_3$ subject to the constraints

$$\begin{matrix} x_1 + x_2 + x_3 \geq 100 \\ 2x_1 + x_3 \geq 50 \end{matrix} \Rightarrow \begin{matrix} x_1 + x_2 + x_3 - s_1 = 100 \\ 2x_1 + x_3 - s_2 = 50 \end{matrix}$$

and $P = -C = -(2x_1 + x_2 + 3x_3) = -2x_1 - x_2 - 3x_3$

$\therefore\ 2x_1 + x_2 + 3x_3 + P = 0$

pivot column

$$\begin{bmatrix} 1 & 1 & 1 & -1 & 0 & 0 & 100 \\ 2 & 0 & 1 & 0 & -1 & 0 & 50 \\ 2 & 1 & 3 & 0 & 0 & 1 & 0 \end{bmatrix} \quad r = \frac{100}{1}, \quad r = \frac{50}{2} = 25 \leftarrow \text{pivot row}$$

$$\frac{1}{2}R_2 \begin{bmatrix} 1 & 1 & 1 & -1 & 0 & 0 & 100 \\ 1 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} & 0 & 25 \\ 2 & 1 & 3 & 0 & 0 & 1 & 0 \end{bmatrix}$$

pivot column

$$\begin{matrix} -R_2 + R_1 \to R_1 \\ \\ -2R_2 + R_3 \to R_3 \end{matrix} \begin{bmatrix} 0 & 1 & \frac{1}{2} & -1 & \frac{1}{2} & 0 & 75 \\ 1 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} & 0 & 25 \\ 0 & 1 & 2 & 0 & 1 & 1 & -50 \end{bmatrix} \quad r = \frac{75}{1} \leftarrow \text{pivot row}$$

$$-R_1 + R_3 \to R_3 \begin{bmatrix} 0 & 1 & \frac{1}{2} & -1 & \frac{1}{2} & 0 & 75 \\ 1 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} & 0 & 25 \\ 0 & 0 & \frac{3}{2} & 1 & 1/2 & 1 & -125 \end{bmatrix}$$

$\therefore$ minimum $C = -P = -(-125) = 125$

at $x_1 = 25,\ x_2 = 75,\ x_3 = 0$

$s_1 = 0,\ s_2 = 0$

Question 2. *(5 marks)* Minimize $C = 2x_1 + x_2 + x_3$ subject to the constraints

$$\begin{aligned} x_1 + x_2 &\geq 30 \\ x_1 + x_2 + 3x_3 &\geq 8 \\ x_1 + x_3 &\geq 1 \end{aligned} \Rightarrow \begin{aligned} x_1 + x_2 - s_1 &= 30 \\ x_1 + x_2 + 3x_3 - s_2 &= 8 \\ x_1 + x_3 - s_3 &= 1 \end{aligned}$$

and $P = -C = -(2x_1 + x_2 + x_3) = -2x_1 - x_2 - x_3$

$\therefore 2x_1 + x_2 + x_3 + P = 0$

pivot column ↓

$$\begin{bmatrix} 1 & 1 & 0 & -1 & 0 & 0 & 0 & 30 \\ 1 & 1 & 3 & 0 & -1 & 0 & 0 & 8 \\ 1 & 0 & 1 & 0 & 0 & -1 & 0 & 1 \\ 2 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$r = 30/1 = 30$; $r = 8$; $r = 1$ ← pivot row

pivot column ↓

$$\begin{aligned} -R_3 + R_1 &\to R_1 \\ -R_3 + R_2 &\to R_2 \\ -2R_3 + R_4 &\to R_4 \end{aligned} \begin{bmatrix} 0 & 1 & -1 & -1 & 0 & 1 & 0 & 29 \\ 0 & 1 & 2 & 0 & -1 & 1 & 0 & 7 \\ 1 & 0 & 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & -1 & 0 & 0 & 2 & 1 & -2 \end{bmatrix}$$

$r = 29/1 = 29$; $r = 7/1 = 7$ ← pivot row

pivot column ↓

$$\begin{aligned} -R_2 + R_1 &\to R_1 \\ -R_2 + R_4 &\to R_4 \end{aligned} \begin{bmatrix} 0 & 0 & -3 & -1 & 1 & 0 & 0 & 22 \\ 0 & 1 & 2 & 0 & -1 & 1 & 0 & 7 \\ 1 & 0 & 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & -3 & 0 & 1 & 1 & 1 & -9 \end{bmatrix}$$

$r = 22/1$ ← pivot row

$$\begin{aligned} R_1 + R_2 &\to R_2 \\ -R_1 + R_2 &\to R_4 \end{aligned} \begin{bmatrix} 0 & 0 & -3 & -1 & 1 & 0 & 0 & 22 \\ 0 & 1 & -1 & -1 & 0 & 1 & 0 & 29 \\ 1 & 0 & 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & -31 \end{bmatrix}$$

$\therefore$ minimum $C = -P = -(-31) = 31$

at $x_1 = 1$, $x_2 = 29$, $x_3 = 0$

$s_1 = 0$, $s_2 = 22$, $s_3 = 0$