Dawson College: **Linear Algebra**: 201-105-DW-S04: Fall 2009

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Bonus Quiz 1

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* Use Cramer's rule solve the following system

$$\begin{array}{rcrcrcr} 2x_1 & + & x_2 & - & 2x_3 & = & -2 \\ & & 3x_2 & + & 3x_3 & = & 1 \\ & & & & x_3 & = & 0 \end{array}.$$

$Ax=b$ where $A = \begin{bmatrix} 2 & 1 & -2 \\ 0 & 3 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

$b = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$

$\det A = 6$

$A_1 = \begin{bmatrix} -2 & 1 & -2 \\ 1 & 3 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

$$\begin{aligned} \det A_1 &= a_{31}C_{31} + a_{32}C_{32} + a_{33}C_{33} \\ &= 0C_{31} + 0C_{32} + 1C_{33} \\ &= (-1)^{3+3}\begin{vmatrix} -2 & 1 \\ 1 & 3 \end{vmatrix} = (1)[-6-1] \\ &= -7 \end{aligned}$$

$A_2 = \begin{bmatrix} 2 & -2 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$ $\det A_2 = 2$

$A_3 = \begin{bmatrix} 2 & -2 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ $\det A_3 = 0$

$\therefore$ $x_1 = \dfrac{\det A_1}{\det A} = \dfrac{-7}{6}$, $x_2 = \dfrac{\det A_2}{\det A} = \dfrac{2}{6} = \dfrac{1}{3}$

$x_3 = \dfrac{\det A_3}{\det A} = \dfrac{0}{6} = 0$

Question 2. *(5 marks)* Use the adjoint to find the inverse of the following matrix

$$B = \begin{bmatrix} 0 & 3 & 4 \\ 0 & 5 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$$

$$B^{-1} = \frac{1}{\det B} \operatorname{Adj}(B) = \frac{1}{\det B} \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^t$$

$$\det B = b_{31} C_{31} + b_{32} C_{32} + b_{33} C_{33}$$
$$= 1 C_{31} + 0 C_{32} + 0 C_{33} = 1(-1)^{3+1} \begin{vmatrix} 3 & 4 \\ 5 & 1 \end{vmatrix} = -17$$

$$C_{11} = (-1)^{1+1} \begin{vmatrix} 5 & 1 \\ 0 & 0 \end{vmatrix} = 0 \qquad C_{21} = (-1)^{2+1} \begin{vmatrix} 3 & 4 \\ 0 & 0 \end{vmatrix} = 0$$

$$C_{12} = (-1)^{1+2} \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = 1 \qquad C_{22} = (-1)^{2+2} \begin{vmatrix} 0 & 4 \\ 1 & 0 \end{vmatrix} = -4$$

$$C_{13} = (-1)^{1+3} \begin{vmatrix} 0 & 5 \\ 1 & 0 \end{vmatrix} = -5 \qquad C_{23} = (-1)^{2+3} \begin{vmatrix} 0 & 3 \\ 1 & 0 \end{vmatrix} = 3$$

$$C_{31} = (-1)^{3+1} \begin{vmatrix} 3 & 4 \\ 5 & 1 \end{vmatrix} = -17, \qquad C_{32} = (-1)^{3+2} \begin{vmatrix} 0 & 4 \\ 0 & 1 \end{vmatrix} = 0$$

$$C_{33} = (-1)^{3+3} \begin{vmatrix} 0 & 3 \\ 0 & 5 \end{vmatrix} = 0$$

$$B^{-1} = \frac{1}{-17} \begin{bmatrix} 0 & 1 & -5 \\ 0 & -4 & 3 \\ -17 & 0 & 0 \end{bmatrix}^t = \frac{1}{-17} \begin{bmatrix} 0 & 0 & -17 \\ 1 & -4 & 0 \\ -5 & 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ \frac{-1}{17} & \frac{4}{17} & 0 \\ \frac{5}{17} & \frac{-3}{17} & 0 \end{bmatrix}$$