Dawson College: **Linear Algrbra**: 201-105-DW-S04: Fall 2009

Name: ____________________
Student ID: ____________________

Test 1

This test is graded out of 49 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(10 marks)* Solve the following system by Gauss-Jordan elimination:

$$\begin{array}{rcrcrcrcrcr} 3x_1 & - & x_2 & + & x_3 & - & 3x_4 & + & x_5 & = & 3 \\ 4x_1 & + & 3x_2 & - & x_3 & + & x_4 & + & x_5 & = & 5 \\ 11x_1 & + & 5x_2 & - & x_3 & & & + & 3x_5 & = & 2 \end{array}$$

Question 2. Consider the matrices:

$$A = \begin{bmatrix} -1 & 2 & 5 \\ 2 & 3 & -6 \\ 1 & 0 & -2 \end{bmatrix}, B = \begin{bmatrix} 3 & 2 & -1 \\ 0 & -4 & 1 \end{bmatrix} C = \begin{bmatrix} 2 & -1 \\ -3 & 4 \\ 0 & -1 \end{bmatrix} D = \begin{bmatrix} 1 & 0 \\ -2 & -2 \end{bmatrix}$$

a. *(2 marks)* Compute the following, if possible.

$$C - D$$

b. *(3 marks)* Compute the following, if possible.

$$BC - D$$

c. *(3 marks)* Compute the following, if possible.

$$C^t B^t$$

d. *(3 marks)* Compute the following, if possible.

$$AB$$

e. *(5 marks)* Find E, if possible.

$$(D^t + I - E^t)^{-1} = D$$

Question 3. *(5 marks)* Given the following augmented matrix in row-echelon form, solve the system using back substitution.

$$\begin{bmatrix} 1 & 2 & 2 & \frac{5}{2} & \frac{1}{2} \\ 0 & 1 & -1 & 3 & 2 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

Question 4. *(3 marks)* Given the following augmented matrix solve the system.

$$\begin{bmatrix} 1 & 2 & 2 & 0 & 1 \\ 0 & 1 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Question 5. *(5 marks)* Find the inverse of

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 5 & 4 \\ -3 & -6 & 10 \end{bmatrix}$$

Question 6. *(5 marks)* Write the matrix

$$\begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$$

as a product of elementary matrices.

Question 7. *(5 marks)* Solve the following system by inverting the coefficient matrix.

$$\begin{array}{rcrcr} -2x_1 & & & = & 1 \\ x_1 & - & x_2 & = & 2 \end{array}$$

Bonus Question. *(3 marks)* Consider the following system:

$$\begin{array}{rcrcrcr} 2x & - & y & + & z & = & 3 \\ x & + & 2y & - & az & = & 22 \\ 4x & + & 3y & - & z & = & b \end{array}$$

where $a, b \in \mathbb{R}$, determine the values of a, b so that the system has

i a unique solution,

ii infitely many solutions,

iii no solutions.