Dawson College: **Linear Algebra**: 201-105-DW-S04: Fall 2009

Name: ____________________
Student ID: ____________________

Test 2

This test is graded out of 47 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Let

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 9 & 3 & 4 & 1 & 0 & 9 \\ 0 & 0 & 1 & 1 & 4 & 9 & 2 & 7 & 7 & 9 \\ 0 & 0 & 0 & 1 & 1 & 4 & 9 & 2 & 7 & 7 \\ 0 & 0 & 0 & 0 & 1 & 1 & 4 & 9 & 2 & 7 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 4 & 9 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 4 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, C = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix} D = \begin{bmatrix} q & r & s & t \\ u & v & w & x \\ y & z & \alpha & \beta \\ \delta & \varepsilon & \gamma & \aleph \end{bmatrix}$$

a. *(2 marks)* Is A invertible, justify.

b. *(3 marks)* If B is a 10×10 matrix show that AB is not invertible.

c. *(5 marks)* If $\det(C) = 2$ and $\det(D) = -3$ then compute $\det(2C^3D^T(3CD)^{-1}C^3)$.

Question 2. *(5 marks)* Use Cramer's rule to solve for x_2 without solving for $x_1,\ x_3,\ x_4$.

$$\begin{array}{ccccccccc} 2x_1 & + & 2x_2 & + & 3x_3 & + & 2x_4 & = & 2 \\ & & x_2 & - & x_3 & + & x_4 & = & 2 \\ & & 2x_2 & - & 3x_3 & - & 3x_4 & = & 0 \\ & & & & & & 5x_4 & = & 0 \end{array}.$$

Question 3. *(5 marks)* If

$$A = \begin{bmatrix} 0 & 0 & 3 & 2 \\ -2 & 3 & -1 & 3 \\ 0 & 2 & 2 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix},$$

compute $\det(A)$ *(use cofactor expansion to compute)*.

Question 4. Let $S = \{1,2,3,4,5,6,7\}$.

a. *(2 marks)* Determine the parity of the permutation $(2,1,3,4,7,6,5)$ of the set S.

b. *(2 marks)* Is $(1,2,3,4,5,6,7)$ a permutation of the set S, justify.

Question 5. *(5 marks)* Consider the matrix:

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}, \; B = \begin{bmatrix} b & 3a & 4b+c \\ e & 3d & 4e+f \\ h & 3g & 4h+i \end{bmatrix}.$$

If $\det(A) = -13$, compute $\det(B)$.

Question 6. Consider the matrix:

$$A = \begin{bmatrix} 2 & 3 & -4 & 0 \\ -1 & 5 & 2 & 1 \\ 3 & 8 & -6 & 2 \\ 0 & 2 & 0 & 3 \end{bmatrix} B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

a. *(2 marks)* Compute $\det(A)$.

b. *(2 marks)* Compute B^{101}.

Question 7. *(7 marks)* Find the area *(use projections)* of the triangle defined by the vertices:

$A(2,-1,3),\ B(-2,1,-3),\ C(7,-5,2).$

Question 8. *(5 marks)* If

$$A = \begin{bmatrix} 2 & 1 & 2 & 4 & 6 \\ 0 & -2 & 1 & -3 & 3 \\ 0 & 2 & 0 & 4 & 3 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 & 3 \end{bmatrix},$$

compute $\det(A)$ *(use elementary operations)*.

Question 9. *(2 marks)* Find all 3×3 diagonal matrices Y that satisfy $Y^2 - 7Y - 10I = 0$.

Bonus Question. *(3 marks)* Establish the identity:

$$||\mathbf{u}+\mathbf{v}||^2 + ||\mathbf{u}-\mathbf{v}||^2 = 2||\mathbf{u}||^2 + 2||\mathbf{v}||^2$$