

Quiz 2

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (5 marks) Consider the matrix:

$$A = \begin{bmatrix} 0 & 3 \\ 4 & 1 \end{bmatrix}$$

a. (3 marks) Compute $A^2 + 2A - I$.

b. (4 marks) Verify that $(A^t)^{-1} = (A^{-1})^t$.

$$\begin{aligned} A^2 + 2A - I &= \begin{bmatrix} 0 & 3 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 0 & 3 \\ 4 & 1 \end{bmatrix} + 2 \begin{bmatrix} 0 & 3 \\ 4 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 12 & 3 \\ 4 & 13 \end{bmatrix} + \begin{bmatrix} 0 & 6 \\ 8 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 11 & 9 \\ 12 & 14 \end{bmatrix} \end{aligned}$$

b)

$$\begin{aligned} \text{LHS} &= (A^t)^{-1} \\ &= \left(\begin{bmatrix} 0 & 3 \\ 4 & 1 \end{bmatrix}^t \right)^{-1} \\ &= \left(\begin{bmatrix} 0 & 4 \\ 3 & 1 \end{bmatrix} \right)^{-1} \\ &= \frac{1}{0(1) - 3(4)} \begin{bmatrix} 1 & -4 \\ -3 & 0 \end{bmatrix} \\ &= \frac{1}{-12} \begin{bmatrix} 1 & -4 \\ -3 & 0 \end{bmatrix} \\ &= \begin{bmatrix} -\frac{1}{12} & \frac{1}{3} \\ \frac{1}{4} & 0 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{RHS} &= (A^{-1})^t \\ &= \left(\begin{bmatrix} 0 & 3 \\ 4 & 1 \end{bmatrix}^{-1} \right)^t \\ &= \left(\frac{1}{0(1) - 3(4)} \begin{bmatrix} 1 & -3 \\ -4 & 0 \end{bmatrix} \right)^t \\ &= \left(\frac{-1}{12} \begin{bmatrix} 1 & -3 \\ -4 & 0 \end{bmatrix} \right)^t \\ &= \begin{bmatrix} -\frac{1}{12} & \frac{1}{4} \\ \frac{1}{3} & 0 \end{bmatrix}^t \\ &= \begin{bmatrix} -\frac{1}{12} & \frac{1}{3} \\ \frac{1}{4} & 0 \end{bmatrix} \end{aligned}$$

∴ RHS = LHS

Question 2.(3 marks) Solve the following system by Gaussian elimination or Gauss-Jordan elimination.

$$\begin{aligned} 2x_1 + x_2 - 2x_3 &= 2 \\ 5x_1 - x_2 + 3x_3 &= 2 \\ 9x_1 + x_2 - x_3 &= 7 \end{aligned}$$

$$\begin{bmatrix} 2 & 1 & -2 & 2 \\ 5 & -1 & 3 & 2 \\ 9 & 1 & -1 & 7 \end{bmatrix}$$

$$\sim \begin{matrix} 2R_2 \\ 2R_3 \end{matrix} \begin{bmatrix} 2 & 1 & -2 & 2 \\ 10 & -2 & 6 & 4 \\ 18 & 2 & -2 & 14 \end{bmatrix}$$

$$\sim \begin{matrix} -5R_1 + R_2 \rightarrow R_2 \\ -9R_1 + R_3 \rightarrow R_3 \end{matrix} \begin{bmatrix} 2 & 1 & -2 & 2 \\ 0 & -7 & 16 & -6 \\ 0 & -7 & 16 & -4 \end{bmatrix}$$

$$\sim \begin{matrix} -R_2 + R_3 \rightarrow R_3 \end{matrix} \begin{bmatrix} 2 & 1 & -2 & 2 \\ 0 & -7 & 16 & -6 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

$$0x_1 + 0x_2 + 0x_3 = 2$$

$$0 = 2$$

inconsistent

∴ no solutions.