

Quiz 3

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Consider the matrix:

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 9 & 3 & 4 & 1 & 0 & 9 \\ 0 & 0 & 1 & 1 & 4 & 9 & 2 & 7 & 7 & 9 \\ 0 & 0 & 0 & 1 & 1 & 4 & 9 & 2 & 7 & 7 \\ 0 & 0 & 0 & 0 & 1 & 1 & 4 & 9 & 2 & 7 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 4 & 9 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 4 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}, C = \begin{bmatrix} -1 & 1 & 2 & 0 \\ 0 & 2 & 3 & 0 \\ 1 & 1 & 0 & 2 \\ 0 & 2 & 0 & 0 \end{bmatrix}$$

- a. (2 marks) Compute $\det(A)$ = product of all entries of main diagonal = 0
 b. (3 marks) Compute $\det(B^{-3})$
 c. (4 marks) Compute $\det(C)$
 d. (1 mark) Find all 3×3 diagonal matrices Y that satisfy $Y^2 - 3Y - 4I = 0$. $\Leftrightarrow (Y - 4I)(Y + I) = 0$

$$b) B^{-3} = \begin{bmatrix} 1^{-3} & 0 & 0 \\ 0 & 3^{-3} & 0 \\ 0 & 0 & 2^{-3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{27} & 0 \\ 0 & 0 & \frac{1}{8} \end{bmatrix}$$

$$\det(B^{-3}) = (1)\left(\frac{1}{27}\right)\left(\frac{1}{8}\right) = \frac{1}{216}$$

$$\begin{aligned} Y &= 4I & Y &= -I \\ &= \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} &= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} c) \det(C) &= C_{14}C_{14} + C_{24}C_{24} + C_{34}C_{34} + C_{44}C_{44} \\ &= 0C_{14} + 0C_{24} + -2C_{34} + 0C_{44} \\ &= -2(-1)^{3+4} \begin{vmatrix} -1 & 1 & 2 \\ 0 & 2 & 3 \\ 0 & 2 & 0 \end{vmatrix} \end{aligned}$$

$$\begin{aligned} &= -2(-1) [C_{31}C_{31} + C_{32}C_{32} + C_{33}C_{33}] \\ &= -2(-1) [0C_{31} + 2C_{32} + 0C_{33}] \end{aligned}$$

$$\begin{aligned} &= -2(-1) 2 (-1)^{3+2} \begin{vmatrix} -1 & 2 \\ 0 & 3 \end{vmatrix} = -2(-1)(2)(-1)(-3) \\ &= 12 \end{aligned}$$