

Quiz 3

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Consider the matrix:

$$A = \begin{bmatrix} 1 & 1 & 1 & 9 & 3 & 4 & 1 & 0 & 9 \\ 0 & 1 & 1 & 4 & 9 & 2 & 7 & 7 & 9 \\ 0 & 0 & 1 & 1 & 4 & 9 & 2 & 7 & 7 \\ 0 & 0 & 0 & 1 & 1 & 4 & 9 & 2 & 7 \\ 0 & 0 & 0 & 0 & 1 & 1 & 4 & 9 & 2 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 4 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 & 2 & 1 \\ 0 & 2 & 3 & 2 \\ -2 & 1 & 0 & -2 \\ 0 & 2 & 0 & 0 \end{bmatrix}$$

a. (2 marks) Compute $\det(A)$ = product of all the entries of main diagonal = 1

b. (3 marks) Compute $\det(B^{-2})$

c. (4 marks) Compute $\det(C)$

d. (1 mark) Find all 3×3 diagonal matrices Y that satisfy $Y^2 - 4Y - 5I = 0 \Leftrightarrow (Y - 5I)(Y + I) = 0$

$$B^{-2} = \begin{bmatrix} (\frac{1}{2})^{-2} & 0 & 0 \\ 0 & (\frac{1}{3})^{-2} & 0 \\ 0 & 0 & 1^2 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Y = 5I \quad Y = -I$$

$$Y = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} \quad Y = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\therefore \det(B^{-2}) = 4(9)(1) = 36$$

$$\det(C) = c_{11}c_{44} + c_{12}c_{42} + c_{13}c_{43} + c_{14}c_{44}$$

$$= 0c_{11} + 2c_{12} + 0c_{13} + 0c_{14}$$

$$= 2(-1)^{4+2} \begin{vmatrix} 0 & 2 & 1 \\ 0 & 3 & 2 \\ -2 & 0 & 2 \end{vmatrix}$$

$$= 2(1) [c_{11}c_{44} + c_{21}c_{42} + c_{31}c_{43}]$$

$$= 2(1) [0c_{11} + 0c_{21} + (-2)c_{31}]$$

$$= 2(1)(-2)(-1) \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix}$$

$$= 2(1)(-2)(1)(4 - 3)$$

$$= -4$$