

Quiz 4

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Consider the matrix:

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}, B = \begin{bmatrix} j & k & l \\ m & n & o \\ p & q & r \end{bmatrix}, C = \begin{bmatrix} 2 & 1 & -2 \\ -3 & 2 & 1 \\ 5 & -2 & -1 \end{bmatrix}, D = \begin{bmatrix} -3 & -3 & 6 \\ 2 & 3 & -4 \\ 5 & 8 & -10 \end{bmatrix}$$

a. (3 marks) If $\det(A) = -7$ and $\det(B) = \frac{1}{3}$ then compute $\det((2B)^t A^2 (3ABA)^{-1})$.

b. (2 marks) Compute $\det(D)$.

c. (5 marks) Compute $\det(C)$ using elementary operations.

$$\begin{aligned} &= \det(2B)^t \det A^2 \det(3ABA)^{-1} \\ &= \det 2B (\det A)^2 \frac{1}{\det(3ABA)} \\ &= 2^3 \det B (\det A)^2 \frac{1}{3^3 \det A \det B \det A} \\ &= \frac{8}{27} \end{aligned}$$

b) $\det(D) = 0$ since $C_3 = -2C_1$

c)

$$C \sim \begin{matrix} 2R_2 \\ 2R_3 \end{matrix} \begin{bmatrix} 2 & 1 & -2 \\ -6 & 4 & 2 \\ 10 & -4 & -2 \end{bmatrix}$$

$$\sim \begin{matrix} 3R_1 + R_2 \rightarrow R_2 \\ -5R_1 + R_3 \rightarrow R_3 \end{matrix} \begin{bmatrix} 2 & 1 & -2 \\ 0 & 7 & -4 \\ 0 & -9 & 8 \end{bmatrix}$$

$$\sim \begin{matrix} 7R_3 \end{matrix} \begin{bmatrix} 2 & 1 & -2 \\ 0 & 7 & -4 \\ 0 & -63 & 56 \end{bmatrix} \sim$$

$$\begin{matrix} 9R_2 + R_3 \rightarrow R_3 \end{matrix} \begin{bmatrix} 2 & 1 & -2 \\ 0 & 7 & -4 \\ 0 & 0 & 20 \end{bmatrix}$$

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$$\begin{aligned} \therefore (2)(2)(7) \det(C) &= \det(E) \\ (2)(2)(7) \det(C) &= 2(7)(20) \\ \det C &= \frac{2(7)(20)}{(2)(2)(7)} \\ \det C &= 10 \end{aligned}$$