

Quiz 4

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Consider the matrix:

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}, B = \begin{bmatrix} j & k & l \\ m & n & o \\ p & q & r \end{bmatrix}, C = \begin{bmatrix} 2 & 3 & -3 \\ 3 & -1 & 2 \\ 5 & 1 & 6 \end{bmatrix}, D = \begin{bmatrix} -3 & -2 & 5 \\ 2 & 3 & -4 \\ 6 & 4 & -10 \end{bmatrix}$$

a. (3 marks) If $\det(A) = -7$ and $\det(B) = \frac{1}{3}$ then compute $\det((3AB)^t B^3 (5BA)^{-1})$.

b. (2 marks) Compute $\det(D)$.

c. (5 marks) Compute $\det(C)$ using elementary operations.

b) $\det D = 0$ since $R_3 = 2R_1$

c)

$$C \sim \begin{matrix} 2R_1 \\ 2R_2 \\ 2R_3 \end{matrix} \begin{bmatrix} 2 & 3 & -3 \\ 6 & -2 & 4 \\ 10 & 2 & 12 \end{bmatrix}$$

$$\begin{aligned} &= \det(3AB)^t \det B^3 \det(5BA)^{-1} \\ &= \det(3AB) (\det B)^3 \frac{1}{\det(5BA)} \\ &= 3^3 \det A \det B (\det B)^3 \frac{1}{5^3 \det B \det A} \\ &= \frac{27 (\frac{1}{3})^3}{125} = \frac{1}{125} \end{aligned}$$

$$\sim \begin{matrix} -3R_1 + R_2 \rightarrow R_2 \\ -5R_1 + R_3 \rightarrow R_3 \end{matrix} \begin{bmatrix} 2 & 3 & -3 \\ 0 & -11 & 13 \\ 0 & -13 & 27 \end{bmatrix} \sim \begin{matrix} -11R_3 \end{matrix} \begin{bmatrix} 2 & 3 & -3 \\ 0 & -11 & 13 \\ 0 & 143 & -297 \end{bmatrix}$$

$$\sim \begin{matrix} 13R_3 + R_3 \rightarrow R_3 \end{matrix} \begin{bmatrix} 2 & 3 & -3 \\ 0 & -11 & 13 \\ 0 & 0 & -128 \end{bmatrix} = E$$

$$\begin{aligned} \therefore (2)(2)(-11) \det C &= \det E_{-64} \\ \det C &= \frac{2(-11)(-128)}{2(2)(-11)} \\ &= -64 \end{aligned}$$