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Quiz 7

This quiz is graded out of 10 marks. No books, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Let $\mathbf{u} = (1,3,-1)$, $\mathbf{v} = (2,-2,-2)$, $\mathbf{w} = (1,-1,-3)$.

a. *(3 marks)* Find the scalar triple product of $\mathbf{u}, \mathbf{v}, \mathbf{w}$.

b. *(1 mark)* Find the volume of the parallelepiped with sides $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$.

$$\vec{u}\cdot(\vec{v}\times\vec{w}) = \begin{vmatrix} 1 & 3 & -1 \\ 2 & -2 & -2 \\ 1 & -1 & -3 \end{vmatrix} = 1(-1)^{1+1}\begin{vmatrix} -2 & -2 \\ -1 & -3 \end{vmatrix} + 3(-1)^{2+1}\begin{vmatrix} 2 & -2 \\ 1 & -3 \end{vmatrix} + (-1)(-1)^{3+1}\begin{vmatrix} 2 & -2 \\ 1 & -1 \end{vmatrix}$$

$$= [6-2] + (-3)[-6+2] + 0$$

$$= 16$$

∴ the volume is 16.

Question 2. *(4 marks)* Find the area of the parallelogram determined by $\mathbf{x} = (3,-2,2)$ and $\mathbf{y} = (-1,1,-3)$.

$$\vec{x}\times\vec{y} = \left(\begin{vmatrix} -2 & 1 \\ 2 & -3 \end{vmatrix}, -\begin{vmatrix} 3 & -1 \\ 2 & -3 \end{vmatrix}, \begin{vmatrix} 3 & -1 \\ -2 & 1 \end{vmatrix}\right) = (4, 7, 1)$$

$3 \quad -1$
$-2 \quad 1$
$2 \quad -3$

∴ the area $= \|\vec{x}\times\vec{y}\| = \|(4,7,1)\| = \sqrt{4^2+7^2+1^2}$

$$= \sqrt{66}$$

Question 3. *(2 marks)* Suppose that $\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w}) = -13$ then find $\mathbf{v}\cdot(\mathbf{w}\times\mathbf{u})$ and justify.

$$\vec{v}\cdot(\vec{w}\times\vec{u}) = -\vec{v}\cdot(\vec{u}\times\vec{w})$$

equivalent to changing two rows of the determinant

$$= -(-1)\,\vec{u}\cdot(\vec{v}\times\vec{w})$$

$$= \vec{u}\cdot(\vec{v}\times\vec{w})$$

$$= -13$$