Name: _____

Student ID: _____

Test 1

This test is graded out of 49 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(10 marks)* Solve the following system by Gauss-Jordan elimination:

$$
\begin{array}{rcrcrcrcr}
3x_1 & + & x_2 & + & x_3 & & & = & 3 \\
4x_1 & + & 3x_2 & + & 3x_3 & + & x_4 & = & 2 \\
5x_1 & + & 5x_2 & + & 5x_3 & + & x_4 & = & 3 \\
7x_1 & + & 4x_2 & + & 4x_3 & + & x_4 & = & 5
\end{array}
$$

Question 2. Consider the matrices:

$$A = \begin{bmatrix} 2 & -1 & 2 \\ 1 & 0 & 3 \\ 2 & 3 & 1 \end{bmatrix}, \; B = \begin{bmatrix} 1 & 0 & -3 \\ 2 & -2 & 1 \end{bmatrix} \; C = \begin{bmatrix} 1 & 0 \\ -2 & 1 \\ 2 & -3 \end{bmatrix} \; D = \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}$$

a. *(2 marks)* Compute the following, if possible.

BD

b. *(4 marks)* Compute the following, if possible.

$CB - 2A$

c. *(4 marks)* Compute the following, if possible.

$\text{tr}(C^T B^T)$

d. *(4 marks)* Find E, if possible.

$(I - E^{-1})^T = D$

Question 3. *(5 marks)* Given the following augmented matrix in row-echelon form, solve the system using back substitution.

$$\begin{bmatrix} 1 & 5 & 2 & 1 & \sqrt{5} \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix}$$

Question 4. *(3 marks)* Find A^{-1} if $A^2 - 2A - I = 0$.

Question 5. *(8 marks)* Solve the following system by inverting the coefficient matrix.

$$
\begin{array}{rcrcrcr}
x_1 & + & 2x_2 & + & 3x_3 & = & 1 \\
-2x_1 & - & 3x_2 & + & 4x_3 & = & 0 \\
3x_1 & + & 6x_2 & + & 10x_3 & = & -1
\end{array}
$$

Question 6. Consider the matrices

$$A = \begin{bmatrix} 3 & 5 \\ 2 & -3 \end{bmatrix}, B = \begin{bmatrix} 2 & -3 \\ 3 & 5 \end{bmatrix}, C = \begin{bmatrix} 3 & 5 \\ 5 & 2 \end{bmatrix}$$

Find the elementary matrices E_1, E_2 and E_3 (if possible) such that

   a. *(2 marks)* $E_1 B = C$

   b. *(2 marks)* $E_2 A = B$

   c. *(2 marks)* $E_3 A = C$

Question 7. *(3 marks)* Show that if A is invertible and $AB = AC$, then $B = C$.

Bonus Question. *(3 marks)* Consider

$$A = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$$

where $a, b, c \neq 0$ and find A^{-1} as a product of elementary matrices.