

1) FIND THE DERIVATIVES OF THE FOLLOWING FUNCTIONS:

a) $f(x) = 5x^2 + 2x^{-2} + \frac{1}{x}$

$g(t) = 2t - 4\sqrt{t} + \frac{3}{\sqrt[3]{t}}$

c) $f(x) = (x^3 + 2x)\left(5 - \frac{1}{x^2}\right)$

d) $f(x) = \frac{\sqrt{x+1}}{x^2+2}$

e) $f(x) = \frac{2}{3}x^3 - (x^2+1)(2x^2-3x+1)$

f) $g(x) = \frac{x}{x^2+1} - \frac{x-1}{x^2-1}$

g) $f(x) = (-x^2 + 6x)^{54}$

h) $h(t) = \frac{t^2 - 2t}{(t+1)(t^2+1)}$

i) $f(x) = \frac{\sqrt{3x-1}}{x^2+1}$

j) $f(x) = (x^2+1)(x^2-3)(5x^2+x)$

k) $f(x) = \sqrt{(x+3)(x^2-1)}$

2) FIND THE EQUATION OF THE TANGENT LINE TO $f(x) = \sqrt{2x+5}$ AT $(2,3)$

3) FIND $h'(2)$ GIVEN

$$h(x) = \frac{x^2 f(x)}{f(x) - g(x)}$$

AND

$$\begin{aligned} f'(2) &= 1 \\ g'(2) &= -1 \\ f(2) &= 3 \\ g(2) &= 2 \end{aligned}$$

DERIVATIVE PROBLEMS cont
SOLUTIONS

$$1) a) f'(x) = 10x - 4x^{-3} - x^{-2} = 10x - \frac{4}{x^3} - \frac{1}{x^2}$$

$$b) g(t) = 2t - 4t^{1/2} + 3t^{-1/3}$$

$$g'(t) = 2 - 2t^{-1/2} - t^{-4/3} = 2 - \frac{2}{t^{1/2}} - \frac{1}{t^{4/3}}$$

$$c) f'(x) = (3x^2 + 2) \left(5 - \frac{1}{x^2}\right) + (x^3 + 2x) \left(0 + 2x^{-3}\right)$$

$$= 15x^2 + 10 - 3 - \frac{2}{x^2} + 2 + \frac{4}{x^2}$$

$$= 15x^2 + \frac{2}{x^2} + 9$$

$$d) f'(x) = \frac{\left(\frac{1}{2}x^{-1/2}\right)(x^2+2) - (x^{1/2}+1)(2x)}{(x^2+2)^2}$$

$$= \frac{\frac{x^2+2}{2x^{1/2}} - \frac{(x^{1/2}+1)(2x)}{1}}{(x^2+2)^2} = \frac{\frac{(x^2+2) - (x^{1/2}+1)(4x^{3/2})}{2x^{1/2}}}{(x^2+2)^2}$$

$$= \frac{\frac{x^2+2 - 4x^2 - 4x^{3/2}}{2x^{1/2}}}{(x^2+2)^2} = \frac{1}{(x^2+2)^2}$$

$$= \frac{-3x^2 - 4x^{3/2} + 2}{2x^{1/2}(x^2+2)^2}$$

$$e) f'(x) = 2x^2 - [(2x)(2x^2 - 3x + 1) + (x^2 + 1)(4x - 3)]$$

$$= 2x^2 - [4x^3 - 6x^2 + 2x + 4x^3 + 4x - 3x^2 - 3]$$

$$= -8x^3 + 11x^2 - 6x + 3$$

$$\begin{aligned}
 f) \quad g'(x) &= \frac{d}{dx} \left[\frac{x}{x^2+1} \right] - \frac{d}{dx} \left[\frac{x-1}{x^2-1} \right] \\
 &= \frac{(1)(x^2+1) - (x)(2x)}{(x^2+1)^2} - \frac{(1)(x^2-1) - (x-1)(2x)}{(x^2-1)^2} \\
 &= \frac{x^2+1-2x^2}{(x^2+1)^2} - \frac{x^2-1-2x^2+2x}{(x^2-1)^2} \\
 &= \frac{-x^2+1}{(x^2+1)^2} - \frac{-x^2+2x-1}{(x^2-1)^2} \\
 &= \frac{(-x^2+1)(x^2-1)^2 - (-x^2+2x-1)(x^2+1)^2}{(x^2+1)^2 (x^2-1)^2}
 \end{aligned}$$

$$g) \quad f'(x) = 54(-x^2+6x)^{53} \cdot \frac{d}{dx}(-x^2+6x)$$

$$= 54(-x^2+6x)^{53} \cdot (-2x+6)$$

$$h) \quad h'(t) = \frac{(2t-2) \cdot (t+1)(t^2+1) - (t^2-2t) \cdot \frac{d}{dx}[(t+1)(t^2+1)]}{[(t+1)(t^2+1)]^2}$$

$$= \frac{(2t-2)(t+1)(t^2+1) - (t^2-2t)[(1)(t^2+1) + (t+1)(2t)]}{(t+1)^2 (t^2+1)^2}$$

$$= \frac{(2t-2)(t+1)(t^2+1) - (t^2-2t)(3t^2+2t+1)}{(t+1)^2 (t^2+1)^2}$$

$$i) f'(x) = \frac{\frac{d}{dx}[(3x-1)^{1/2}] \cdot (x^2+1) - (3x-1)^{1/2} \cdot \frac{d}{dx}(x^2+1)}{(x^2+1)^2}$$

$$= \frac{\frac{1}{2}(3x-1)^{-1/2}(x^2+1) - (3x-1)^{1/2}(2x)}{(x^2+1)^2}$$

$$= \frac{\frac{x^2+1}{2(3x-1)} - \frac{(3x-1)^{1/2}(2x)}{1}}{(x^2+1)^2}$$

$$= \frac{(x^2+1) - (3x-1)(4x)}{2(3x-1)} \cdot \frac{1}{(x^2+1)^2}$$

$$= \frac{-11x^2 + 4x + 1}{2(3x-1)(x^2+1)^2}$$

$$j) f'(x) = \frac{d}{dx}[(x^2+1)(x^2-3)](5x^2+x) + (x^2+1)(x^2-3) \frac{d}{dx}(5x^2+x)$$

$$= [(2x)(x^2-3) + (x^2+1)(2x)](5x^2+x) + (x^2+1)(x^2-3)(10x+1)$$

$$= (2x^3-6x)(5x^2+x) + (2x^3+2x)(5x^2+x) + (x^2+1)(x^2-3)(10x+1)$$

$$k) f'(x) = \frac{1}{2}((x+3)(x^2-1))^{-1/2} \cdot \frac{d}{dx}[(x+3)(x^2-1)]$$

$$= \frac{1}{2\sqrt{(x+3)(x^2-1)}} \cdot [(1)(x^2-1) + (x+3)(2x)]$$

$$= \frac{x^2-1 + 2x^2+6x}{2\sqrt{(x+3)(x^2-1)}} = \frac{3x^2+6x-1}{2\sqrt{(x+3)(x^2-1)}}$$

$$\begin{aligned}
 2) \quad f'(x) &= \frac{1}{2} (2x+5)^{-1/2} \cdot \frac{d}{dx} (2x+5) \\
 &= \frac{1}{2\sqrt{2x+5}} \cdot 2 \\
 &= \frac{1}{\sqrt{2x+5}}
 \end{aligned}$$

THE SLOPE OF THE TANGENT LINE AT $(2, 3)$ IS GIVEN BY

$$f'(2) = \frac{1}{\sqrt{2(2)+5}} = \frac{1}{\sqrt{9}} = \frac{1}{3}$$

SO THE TANGENT LINE IS:

$$y = ax + b$$

$$3 = \frac{1}{3}(2) + b$$

$$3 - \frac{2}{3} = b$$

$$\frac{7}{3} = b$$

$$\therefore \boxed{y = \frac{1}{3}x + \frac{7}{3}}$$

$$\begin{aligned}
 3) \quad h'(x) &= \frac{\frac{d}{dx} [x^2 f(x)] (f(x) - g(x)) - x^2 f(x) \frac{d}{dx} [f(x) - g(x)]}{(f(x) - g(x))^2} \\
 &= \frac{(2x f(x) + x^2 f'(x)) \cdot (f(x) - g(x)) - x^2 f(x) \cdot (f'(x) - g'(x))}{(f(x) - g(x))^2}
 \end{aligned}$$

AND SO

$$h'(2) = \frac{[2(2)f(2) + (2)^2 f'(2)][f(2) - g(2)] - (2)^2 f(2)[f'(2) - g'(2)]}{[f(2) - g(2)]^2}$$

$$= \frac{[4(3) + 4(1)][3 - 2] - 4(3)[1 - (-1)]}{[3 - 2]^2}$$

$$= \frac{(12 + 4)(1) - 12(2)}{1^2}$$

$$= -8$$