Dawson College: **Calculus III**: 201-BZF-05 S01 September 18, 2012

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Quiz 2

Question 1. *(10 marks)* Determine if the following SERIES converge or diverge:

(a) $\sum_{n=1}^{\infty} \frac{5+2n}{(1+n^2)^2}$ LET $a_n = \frac{5+2n}{(1+n^2)^2}$ AND $b_n = \frac{1}{n^3}$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{5+2n}{(1+n^2)^2} \cdot \frac{n^3}{1} = \lim_{n\to\infty} \frac{5n^3+2n^4}{(1+n^2)^2} = \lim_{n\to\infty} \frac{5/n+2}{\frac{(1+n^2)^2}{n^4}}$$

$$= \lim_{n\to\infty} \frac{5/n+2}{\left(\frac{1}{n^2}+1\right)^2} = \frac{0+2}{(0+1)^2} = 2 > 0$$

NOW, $\sum_{n=1}^{\infty} \frac{1}{n^3}$ CONVERGES (P-SERIES, $P=3$)

$\therefore \sum_{n=1}^{\infty} \frac{5+2n}{(1+n^2)^2}$ CONVERGES BY LIMIT COMPARISON TEST

(b) $\sum_{n=1}^{\infty} \frac{e^{-1/n}}{n^2}$ LET $f(x) = \frac{e^{-1/x}}{x^2}$ THEN f IS POSITIVE AND CONTINUOUS ON $[1,\infty)$

$$f'(x) = \frac{e^{-1/x} \cdot \left(\frac{1}{x^2}\right) \cdot x^2 - 2x\left(e^{-1/x}\right)}{x^4} = \frac{e^{-1/x}(1-2x)}{x^4} < 0$$

WHEN $x \geq 1$ SO f IS DECREASING

NOW, $\int_1^{\infty} \frac{e^{-1/x}}{x^2}\,dx = \lim_{b\to\infty} \int_1^b \frac{e^{-1/x}}{x^2}\,dx = I$

$$\int \frac{e^{-1/x}}{x^2}\,dx = \int e^u\,du = e^u + C = e^{-1/x} + C$$

LET $u = -1/x$
$du = 1/x^2\,dx$

$$\therefore I = \lim_{b\to\infty} \left[e^{-1/x}\right]_1^b = \lim_{b\to\infty}\left(e^{-1/b} - e^{-1}\right) = e^0 - e^{-1} = 1 - \frac{1}{e}$$

SINCE THE IMPROPER INTEGRAL CONVERGES THE SERIES CONVERGES BY INTEGRAL TEST.