Last Name: ___SOLUTIONS___

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Quiz 6

Question 1. Find the derivatives of the following functions. You may use any method but remember to use correct notation.

(a) *(3 marks)* $f(t) = \sqrt[3]{1 + \tan t} = (1 + \tan t)^{1/3}$

$$f'(t) = \frac{1}{3}(1 + \tan t)^{-2/3} \cdot \frac{d}{dt}[1 + \tan t]$$

$$= \frac{1}{3}(1 + \tan t)^{-2/3} \cdot \sec^2 t$$

(b) *(3 marks)* $y = \cot^2(\sin \theta) = \left[\cot(\sin \theta)\right]^2$

$$y' = 2\cot(\sin \theta) \cdot \frac{d}{d\theta}\left[\cot(\sin \theta)\right]$$

$$= 2\cot(\sin \theta) \cdot \left[-\csc^2(\sin \theta) \cdot \frac{d}{d\theta}(\sin \theta)\right]$$

$$= -2\cot(\sin \theta) \cdot \csc^2(\sin \theta) \cdot \cos \theta$$

(c) *(4 marks)* *(Simplify your answer.)* $y = \left(\dfrac{x^2+1}{x^2-1}\right)^3$

$$\frac{dy}{dx} = 3\left(\frac{x^2+1}{x^2-1}\right)^2 \cdot \frac{d}{dx}\left[\frac{x^2+1}{x^2-1}\right] = 3\left(\frac{x^2+1}{x^2-1}\right)^2 \cdot \frac{\frac{d}{dx}[x^2+1](x^2-1) - (x^2+1)\frac{d}{dx}(x^2-1)}{(x^2-1)^2}$$

$$= 3\left(\frac{x^2+1}{x^2-1}\right)^2 \cdot \frac{2x(x^2-1) - (x^2+1)(2x)}{(x^2-1)^2}$$

$$= 3\left(\frac{x^2+1}{x^2-1}\right)^2 \cdot \frac{2x\left[x^2-1-x^2-1\right]}{(x^2-1)^2}$$

$$= 3\left(\frac{x^2+1}{x^2-1}\right)^2 \cdot \frac{2x(-2)}{(x^2-1)^2}$$

$$= \frac{-12x(x^2+1)^2}{(x^2-1)^4}$$

$$= \frac{-12x(x^2+1)^2}{(x+1)^4 (x-1)^4}$$