Last Name: SOLUTIONS

First Name: _____

Student ID: _____

Quiz 9

Question 1. *(5 marks)* Evaluate the following limit (show all of your work).

$\lim_{x\to 0^+} (\tan 2x)^x$ LET $y = (\tan 2x)^x \Rightarrow \ln y = x \ln(\tan 2x)$

$\therefore \lim_{x\to 0^+} x\ln(\tan 2x) = \text{I.F. } 0\cdot\infty = \lim_{x\to 0^+} \frac{\ln(\tan 2x)}{1/x} = \text{I.F. } \frac{\infty}{\infty}$

$\overset{(H)}{=} \lim_{x\to 0^+} \frac{\frac{1}{\tan 2x}\cdot \sec^2 2x \cdot 2}{-1/x^2} = \lim_{x\to 0^+} \frac{-2x^2 \sec^2 2x}{\tan 2x} = \text{I.F. } \frac{0}{0}$

$\overset{(H)}{=} \lim_{x\to 0^+} \frac{-4x\sec^2 2x - 2x^2 \cdot 2\sec 2x \cdot \sec 2x \tan 2x \cdot 2}{\sec^2 2x \cdot 2}$

$= \frac{0}{2} = 0$

$\therefore \lim_{x\to 0^+} (\tan 2x)^x = \lim_{x\to 0^+} y = \lim_{x\to 0^+} e^{\ln y}$

$= e^{\lim_{x\to 0^+} \ln y} = e^0 = 1$

Question 2. *(5 marks)* Find the critical numbers of the function:

$$g(y) = \frac{y-1}{y^2-y+1}$$

$$g'(y) = \frac{(1)(y^2-y+1) - (y-1)(2y-1)}{(y^2-y+1)^2}$$

$$= \frac{y^2-y+1-(2y^2-y-2y+1)}{(y^2-y+1)^2}$$

$$= \frac{y^2-y+1-2y^2+3y-1}{(y^2-y+1)^2}$$

$$= \frac{-y^2+2y}{(y^2-y+1)^2}$$

$g'(y)=0$

$-y^2+2y=0$

$y(-y+2)=0$

$\therefore y=0,2$

$g'(y)$ D.N.E.

$(y^2-y+1)^2=0$

BUT THIS DOESN'T GIVE US ANY C.N. FOR TWO REASONS!

1) IF $y^2-y+1=0$ THEN $g(y)$ IS UNDEFINED

2) $y^2-y+1=0$ HAS NO SOLUTIONS

$$y = \frac{-(-1) \pm \sqrt{(-1)^2-4(1)(1)}}{2(1)}$$ NO SOLUTIONS

$\therefore y=0$ AND $y=2$ ARE THE CRITICAL NUMBERS OF g.