Dawson College: **Linear Algebra**: 201-105-dw-S03: Fall 2012

Name: ____________________
Student ID: ____________________

Test 2

This test is graded out of 50 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Given

$$P_1(-1,2,1),\ P_2(-2,-1,1),\ \mathbf{u}=(-1,2,-3),\ \mathbf{v}=(3,1,4),\ \mathbf{w}=(5,-1,-2)$$

a. *(2 marks)* Sketch the vector $\overrightarrow{P_1P_2}$ with the initial point located at the origin.

b. *(4 marks)* Find the angle θ in radians between $\mathbf{u}$ and $\mathbf{v}$.

c. *(4 marks)* Find two unit vectors orthogonal to $\mathbf{w}$.

Question 2. *(2 marks)* Show that if A is a square matrix, then

$$\det(A^T A) = \det(AA^T)$$

Question 3. *(2 marks)* Prove or disprove*(using an example)*: If A and B are square matrices then

$$\det(A+B) = \det(A) + \det(B)$$

Question 3. *(5 marks)* Solve using Cramer's rule.

$$\begin{array}{rcrcrcr} 3x_1 & - & x_2 & + & x_3 & = & 3 \\ x_1 & + & 3x_2 & - & 5x_3 & = & 1 \\ -2x_1 & - & x_2 & - & x_3 & = & -2 \end{array}$$

Question 4. *(4 marks)* Let A and C be invertible matrices of the same size such that $\det(A) = 4$. Find $\det(C)$ if, $A^2 = C^2A^T$.

Question 5. *(4 marks)* Let A and B be 5×5 matrices such that $\det(A) = 2$, $\det(B) = 3$. Find $\det\left(\det(A)B^2 - 3B^2\right)$.

Question 6. *(5 marks)* Evaluate the determinant

$$\begin{vmatrix} 5d & -a & 4g-7a \\ 5e & -b & 4h-7b \\ 5f & -c & 4i-7c \end{vmatrix}$$

given that

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 2$$

Question 7. Given

$$A = \begin{bmatrix} 1 & 3 & 1 & 5 & 3 \\ -2 & -7 & 0 & -4 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} -6 & 10 & -8 \\ -7 & 1 & -2 \\ 3 & -5 & 4 \end{bmatrix}$$

a. *(5 marks)* Is A invertible, justify.

b. *(2 marks)* Is B invertible, justify.

Question 8. *(5 marks)* Solve for x

$$\begin{vmatrix} x & -1 \\ 3 & 1-x \end{vmatrix} = \begin{vmatrix} 1 & 0 & -3 \\ 2 & x & -6 \\ 1 & 3 & x-5 \end{vmatrix}$$

Question 9. *(6 marks)* Decide whether the given matrix is invertible, and if so, use the adjoint method to find its inverse.

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 3 & -1 & 0 \\ -2 & 5 & 6 \end{bmatrix}$$

Bonus Question. *(5 marks)* Show that

$$\det(A) = \frac{1}{2}\begin{vmatrix} \operatorname{tr}(A) & 1 \\ \operatorname{tr}(A^2) & \operatorname{tr}(A) \end{vmatrix}$$

for every 2×2 matrix A.