Dawson College: **Linear Algebra**: 201-105-dw-S04: Fall 2012

Name: ____________________
Student ID: ____________________

Test 2

This test is graded out of 50 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Let

$$A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \; B = \begin{bmatrix} 3 & 0 \\ 0 & 3 \\ 3 & 0 \end{bmatrix}$$

a. *(1 mark)* Compute the determinant of A.

b. *(4 marks)* Determine the matrix of cofactors of A.

c. *(1 mark)* Determine $\text{adj}(A)$.

d. *(1 mark)* Determine A^{-1}.

e. *(2 marks)* Solve

$$AX = B$$

for X using the inverse of A.

Question 2. *(2 marks)* Find the value(s) of λ in order for A to be symmetric

$$A = \begin{bmatrix} 1 & 7 \\ \lambda^2 - 2 & -3 \end{bmatrix}$$

Question 3. *(2 marks)* Find all 5×5 diagonal matrices A that satisfy

$$A^2 - 7A + 10I$$

Question 4. *(5 marks)* Find all values of λ for which $\det(A) = 0$.

$$\begin{bmatrix} \lambda - 4 & \lambda^{101} & 5 & 0 \\ -1 & 0 & \lambda & 0 \\ 0 & \lambda + 2 & 0 & 0 \\ 1 & 2 & 3 & \lambda \end{bmatrix}$$

Question 5. *(6 marks)* Solve *only* for x_2 using Cramer's rule.

$$\begin{array}{rcrcrcrcrcr}
3x_1 & - & x_2 & + & x_3 & - & 3x_4 & + & 4x_5 & = & 2 \\
 & & 3x_2 & & & & & - & 10x_5 & = & 3 \\
 & & & - & x_3 & - & 2x_4 & + & 3x_5 & = & -3 \\
 & & & & & - & x_4 & - & 3x_5 & = & 6 \\
 & & & & & & & & 4x_5 & = & 9
\end{array}$$

Question 6. *(5 marks)* Evaluate the determinant

$$\begin{vmatrix} d & f & e \\ 2a & 2c & 2b \\ g-4d & i-4f & h-4e \end{vmatrix}$$

given that

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 2$$

Question 7. *(4 marks)* Prove that a square matric A is invertible if and only if $A^T A$ is invertible.

Question 8. *(5 marks)* Find the determinant given that A and B are 7×7 matrices for which $\det(\mathrm{A}) = 3$ and $\det(\mathrm{B}) = 5$. *Justify all your work, as usual*

$$\det((2AB^T)^{-1}(3B)A^3)$$

Question 10. Given

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 9 & 3 & 4 & 1 & 0 & 9 \\ 0 & 0 & 1 & 1 & 4 & 9 & 2 & 7 & 7 & 9 \\ 0 & 0 & 0 & 1 & 1 & 4 & 9 & 2 & 7 & 7 \\ 0 & 0 & 0 & 0 & 1 & 1 & 4 & 9 & 2 & 7 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 4 & 9 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 4 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} -2 & 3 & 2 & 6 \\ 1 & 2 & 3 & -3 \\ -3 & 0 & 1 & 9 \\ 4 & 5 & 5 & -12 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$$

a. *(2 marks)* Is A invertible, justify.

b. *(2 marks)* Is B invertible, justify.

c. *(2 marks)* Compute the determinant of C.

Question 10. Given

$$P_1(-1,0,1),\ P_2(0,-1,0),\ \mathbf{u}=(1,-2,3),\ \mathbf{v}=(-3,5,1),\ \mathbf{w}=(5,1,-2)$$

a. *(2 marks)* Sketch the vector $\overrightarrow{P_1P_2}$ with the initial point located at the origin.

b. *(2 marks)* Find the initial point of the vector that is equivalent to **u** and whose terminal point is P_1.

c. *(2 marks)* Determine $(2\mathbf{u}-7\mathbf{w})-(8\mathbf{v}+\mathbf{u})$.

Bonus Question. *(5 marks)* Given

$$\begin{array}{rcrcl} b\cos\gamma & + & c\cos\beta & = & a \\ c\cos\alpha & + & a\cos\gamma & = & b \\ a\cos\beta & + & b\cos\alpha & = & c \end{array}$$

apply Cramer's rule to show that

$$\cos\alpha=\frac{b^2+c^2-a^2}{2bc}$$