Dawson College: **Linear Algebra**: 201-105-dw-S04: Fall 2012

Name: _____
Student ID: _____

Test 3

This test is graded out of 50 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Given the following vertices $A(1,1,0)$, $B(-2,2,1)$, $C(3,-3,-2)$.

a. *(3 marks)* Find the area of the triangle ΔABC using the cross-product.

b. *(3 marks)* Find the perimeter of the triangle ΔABC.

c. *(3 marks)* Find the length of the altitude from vertex B to side AC using projections.

b. *(1 mark)* Find the area of the triangle ΔABC using part c.

Question 2. Given

$$
\begin{array}{rrcll}
\mathscr{L}_1: & (x,y,z) & = & (2+5t,1+t,-t) & t\in\mathbb{R} \\
\mathscr{L}_2: & (x,y,z) & = & (7+2t,4,10t) & t\in\mathbb{R} \\
\mathscr{L}_3: & (x,y,z) & = & (9-t,2,9-5t) & t\in\mathbb{R} \\
\mathscr{P}_1: & x-2y+3z-11 & = & 0 & \\
\mathscr{P}_2: & -5x-y+z+31 & = & 0 & \\
\mathscr{P}_3: & -3x+6y-9z+1 & = & 0 &
\end{array}
$$

a. *(2 marks)* Are $\mathscr{P}_1$ and $\mathscr{L}_2$ parallel, perpendicular, or neither, justify?

b. *(2 marks)* Are $\mathscr{P}_1$ and $\mathscr{P}_3$ parallel, perpendicular, or neither, justify?

c. *(2 marks)* Are $\mathscr{P}_2$ and $\mathscr{P}_3$ parallel, perpendicular, or neither, justify?

d. *(2 marks)* Are $\mathscr{L}_2$ and $\mathscr{L}_3$ parallel, perpendicular, or neither, justify?

e. *(3 marks)* Find the point intersection between $\mathscr{P}_1$ and $\mathscr{L}_1$ if it exists.

Question 3. *(5 marks)* Suppose that the initial point of the vector $\mathbf{u} = (1,0,-3)$ lies on the line $(x,y,z) = (-1+2t, t, 9-3t)$ where $t \in \mathbb{R}$. Find the angle in radians between the vector and the line.

Question 4. *(5 marks)* Find the distance between the following two parallel planes

$$\begin{array}{lrcl} \mathscr{P}_1: & x-2y+3z-1 & = & 0 \\ \mathscr{P}_2: & -x+2y-3z+3 & = & 0. \end{array}$$

Do NOT use the formula: $D = \frac{|ax_0+by_0+cz_0+d|}{\sqrt{a^2+b^2+c^2}}$

Question 5. Given

$$\begin{aligned} \mathscr{P}_1: \quad & x-3y+3z-1 &=& \ 0 \\ \mathscr{P}_2: \quad & -2x+2y-3z+3 &=& \ 0. \end{aligned}$$

a. *(5 marks)* Find the intersection of $\mathscr{P}_1$ and $\mathscr{P}_2$.

b. *(3 marks)* Find the equation of the plane perpendicular to the intersection of $\mathscr{P}_1$ and $\mathscr{P}_2$ and that passes through $P(1,2,-3)$.

Question 6. *(5 marks)* Find the point of intersection of $(x,y,z) = (1+2t, -2-3t, 3+5t)$ where $t \in \mathbb{R}$ and $(x,y,z) = (2-s, 1+2s, 19-s)$ where $s \in \mathbb{R}$.

Question 7.

a. *(2 marks)* If $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = 2$ then find a scalar triple product of $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ which is equal to -2. Justify.

b. *(2 marks)* Find a vector of length 3 which is oppositely directed to $\mathbf{w} = (1, -3, 2)$.

c. *(2 marks)* For which values of m and n the vectors $\mathbf{u} = (15, m, 1)$ and $\mathbf{v} = (18, 12, n)$ are parallel.

Bonus. *(5 marks)* Find the distance between

$$\begin{array}{lllll} \mathscr{L}_1\colon & (x,y,z) & = & (2+5t,1+t,-t) & t\in\mathbb{R} \\ \mathscr{L}_2\colon & (x,y,z) & = & (7+2t,4,10t) & t\in\mathbb{R} \end{array}$$