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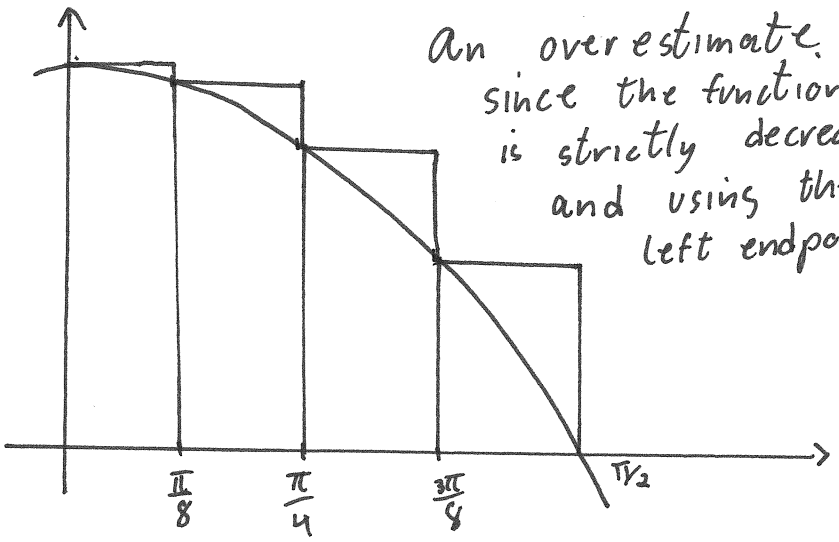
Quiz 3

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(6 marks)* §5.1 #3b Estimate the area under the graph of $f(x) = \cos x$ from $x = 0$ to $x = \frac{\pi}{2}$ using four approximating rectangles and left endpoints. Sketch the graph and the rectangles. Is your estimate and underestimate or an overestimate?



An overestimate since the function is strictly decreasing and using the left endpoint.

$$\Delta x = \frac{b-a}{n} = \frac{\frac{\pi}{2}-0}{4} = \frac{\pi}{8}$$

$$x_i = a + i\Delta x = \frac{i\pi}{8}$$

$x_0 = 0$
$x_1 = \frac{\pi}{8}$
$x_2 = \frac{\pi}{4}$
$x_3 = \frac{3\pi}{8}$
$x_4 = \frac{\pi}{2}$

$$\text{Area} \approx \sum_{i=1}^{4} f(x_{i-1})\Delta x$$

$$= \frac{\pi}{8}\left[f(x_0) + f(x_1) + f(x_2) + f(x_3)\right]$$

$$= \frac{\pi}{8}\left[f(0) + f(\tfrac{\pi}{8}) + f(\tfrac{\pi}{4}) + f(\tfrac{3\pi}{8})\right]$$

$$= \frac{\pi}{8}\left[\cos 0 + \cos(\tfrac{\pi}{8}) + \cos(\tfrac{\pi}{4}) + \cos(\tfrac{3\pi}{8})\right]$$

$$= \frac{\pi}{8}\left[1 + \cos(\tfrac{\pi}{8}) + \frac{1}{\sqrt{2}} + \cos(\tfrac{3\pi}{8})\right]$$

Question 2. *(4 marks)* §5.1 #17 Determine a region whose area is equal to

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi}{4n} \tan\frac{i\pi}{4n} = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

$\Delta x = \frac{\pi}{4n}$

$\Delta x = \frac{b-a}{n}$

$\frac{\pi}{4n} = \frac{b-a}{n}$

$\frac{\pi}{4} = b - a$

$x_i = a + i\Delta x$

$x_i = i\frac{\pi}{4n}$

$\therefore a = 0$ and $b = \frac{\pi}{4}$

$f(x) = \tan x$