Linear Algebra Dawson College
201-NYC-05 01 Math Department
Test 2 March 31, 2014

[6] **1.** Find and describe the general solution of the following system using Gauss-Jordan elimination.

$$\begin{cases} x_1 + 3x_2 - 2x_3 + 5x_4 - 3x_5 = 1 \\ 2x_1 + 7x_2 - 3x_3 + 7x_4 - 5x_5 = 2 \\ 3x_1 + 11x_2 - 4x_3 + 10x_4 - 9x_5 = 3 \end{cases}$$

[6] **2.** For what value(s) of t will the following system have **i)** no solution, **ii)** one solution, and **iii)** infinitely many solutions?

$$\begin{cases} 6x - y + z = 5 \\ tx + z = 1 \\ y + tz = -t \end{cases}$$

3. Let $A = \begin{pmatrix} 1 & -2 & 3 \\ 2 & -5 & 10 \\ -1 & 2 & -2 \end{pmatrix}$

[6] **a)** Find A^{-1} by row-reduction.

[4] **b)** Using A^{-1} from part **a)**, solve the system $A\mathbf{x} = \mathbf{b}$, where $\mathbf{b} = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix}$.

[4] **c)** Write A as a product of elementary matrices.

[4] **4.** If $A = \begin{pmatrix} -1 & 0 & 2 \\ 3 & 4 & 5 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 2 & 4 \\ -1 & 3 & 0 \\ 4 & 5 & 6 \end{pmatrix}$, and $C = \begin{pmatrix} 2 & 0 \\ 0 & -1 \\ 3 & 4 \end{pmatrix}$, find $C^T B A^T$.

[3] **5.** Write $\begin{pmatrix} -1 & 2 & -3 \\ 0 & 4 & 5 \\ 1 & -5 & 0 \end{pmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

[4] **6.** Given $A = \begin{pmatrix} -2 & -3 & -7 \\ 1 & -1 & -4 \\ 0 & 1 & 3 \end{pmatrix}$, show that $A^3 = 0$. Use a formula proved in class to find the inverse of $I - A$.

[4] **7.** If a, b, and c are not equal, show that the rank of $\begin{pmatrix} 1 & a & b+c \\ 1 & b & a+c \\ 1 & c & a+b \end{pmatrix}$ is 2.

[4] **8.** Suppose A is a 3×3 matrix whose rank is 2.

a) Does the system $A\mathbf{x} = \mathbf{0}$ have a non-trivial solution? Explain.

b) Is the system $A\mathbf{x} = \mathbf{b}$ consistent for every possible $\mathbf{b}$? Explain.

[*3*] **9.** Define a *square root* of a square matrix A as any matrix B such that $B^2 = A$. Show that for any real number k, the matrix $\begin{pmatrix} k & 1+k \\ 1-k & -k \end{pmatrix}$ is a square root of I_2. (So that I_2 has an infinite number of square roots!!)

[*4*] **10.** If A is a symmetric $n \times n$ matrix and P is any $m \times n$ matrix, show that PAP^T is symmetric.

[*4*] **11.** If $AB = \lambda B$, where A is $n \times n$, B is $n \times p$, and λ is a scalar, show that $A^m B = \lambda^m B$ for any positive integer m.

[*Total Points = 56*]

[*3*] **B1.** Let $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 2 & 3 \\ 1 & -1 & 3 \end{pmatrix}$, $C = \begin{pmatrix} 2 & 1 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}$, and $\mathbf{d} = \begin{pmatrix} 10 \\ 13 \\ 9 \end{pmatrix}$. Given that $C\mathbf{b} = \mathbf{d}$, solve the system $A\mathbf{x} = \mathbf{b}$.

[*3*] **B2.** Suppose A is a 3×4 matrix satisfying $A \begin{pmatrix} 1 \\ 2 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $A \begin{pmatrix} 0 \\ 3 \\ 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

Find a 4×1 matrix $\mathbf{x}$ such that $A\mathbf{x} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$. Justify your answer.