Dawson College: Linear Algebra (SCIENCE): 201-NYC-05-S6: Fall 2015

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Quiz 10

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §4.2 #2b Use the Subspace Test to determine which of the following are subspaces of $\mathcal{M}_{n\times n}$.
The set of all $n\times n$ matrices A such that $\det(A) = 0$.

$$A = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \in \{M \mid M \in \mathcal{M}_{n\times n} \text{ and } |M| = 0\}$$

$A + B = I_n$ and $\det(A+B) = \det(I_n) = 1$

$\therefore A+B \notin \{M \mid M \in \mathcal{M}_{n\times n} \text{ and } |M| = 0\}$ $\therefore$ not closed under addition

$\therefore$ not a subspace since it fails the subspace test.

Question 2. §4.2 #12 Suppose that $\vec{v}_1 = (2,1,0,3)$, $\vec{v}_2 = (3,-1,5,2)$, and $\vec{v}_3 = (-1,0,2,1)$. Is the following vector in $\operatorname{span}(\vec{v}_1, \vec{v}_2, \vec{v}_3)$.

$(1,1,1,1) = c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3$

$(1,1,1,1) = c_1(2,1,0,3) + c_2(3,-1,5,2) + c_3(-1,0,2,1)$

$$\begin{aligned} 2c_1 + 3c_2 - c_3 &= 1 \\ c_1 - c_2 &= 1 \\ 5c_2 + 2c_3 &= 1 \\ 3c_1 + 2c_2 + c_3 &= 1 \end{aligned} \qquad \begin{bmatrix} 2 & 3 & -1 & 1 \\ 1 & -1 & 0 & 1 \\ 0 & 5 & 2 & 1 \\ 3 & 2 & 1 & 1 \end{bmatrix} \sim \begin{array}{c} R_1 \leftrightarrow R_2 \\ \\ \\ \\ \end{array} \begin{bmatrix} 1 & -1 & 0 & 1 \\ 2 & 3 & -1 & 1 \\ 0 & 5 & 2 & 1 \\ 3 & 2 & 1 & 1 \end{bmatrix}$$

$$\sim \begin{array}{c} \\ -2R_1+R_2 \to R_2 \\ \\ -3R_1+R_4 \to R_4 \end{array} \begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & 5 & -1 & -1 \\ 0 & 5 & 2 & 1 \\ 0 & 5 & 1 & -2 \end{bmatrix} \sim \begin{array}{c} \\ \\ -R_2+R_3 \to R_3 \\ -R_2+R_4 \to R_4 \end{array} \begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & 5 & -1 & -1 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 2 & -1 \end{bmatrix}$$

$$\sim \begin{array}{c} \\ \\ \\ -\frac{2}{3}R_3+R_4 \to R_4 \end{array} \begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & 5 & -1 & -1 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & -7/3 \end{bmatrix}$$

$\therefore$ inconsistent

$\therefore (1,1,1,1) \notin \operatorname{span}(\{\vec{v}_1, \vec{v}_2, \vec{v}_3\})$