Dawson College: Linear Algebra (SCIENCE): 201-NYC-05-S6: Fall 2015

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Quiz 11

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §4.3 #9 *(5 marks)* For which real values of λ do the following vectors form a linearly dependent set in $\mathbb{R}^3$?

$$\vec{v}_1 = (\lambda, -\frac{1}{2}, -\frac{1}{2}),\ \vec{v}_2 = (-\frac{1}{2}, \lambda, -\frac{1}{2}),\ \vec{v}_3 = (-\frac{1}{2}, -\frac{1}{2}, \lambda)$$

$$\vec{0} = c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3$$

$$(0,0,0) = c_1(\lambda, -\tfrac{1}{2}, -\tfrac{1}{2}) + c_2(-\tfrac{1}{2}, \lambda, -\tfrac{1}{2}) + c_3(-\tfrac{1}{2}, -\tfrac{1}{2}, \lambda)$$

$$\underbrace{\begin{bmatrix} \lambda & -1/2 & -1/2 \\ -1/2 & \lambda & -1/2 \\ -1/2 & -1/2 & \lambda \end{bmatrix}}_{A} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly dependent iff $|A| = 0$

$$|A| = \lambda \begin{vmatrix} \lambda & -1/2 \\ -1/2 & \lambda \end{vmatrix} + 1/2 \begin{vmatrix} -1/2 & -1/2 \\ -1/2 & \lambda \end{vmatrix} - \frac{1}{2} \begin{vmatrix} -1/2 & \lambda \\ -1/2 & -1/2 \end{vmatrix}$$

$$= \lambda(\lambda^2 - 1/4) + \frac{1}{2}\left(-\frac{\lambda}{2} - \frac{1}{4}\right) - \frac{1}{2}\left(\frac{1}{4} + \frac{\lambda}{2}\right)$$

$$= \lambda(\lambda - \tfrac{1}{2})(\lambda + \tfrac{1}{2}) - \tfrac{1}{4}(\lambda + \tfrac{1}{2}) - \tfrac{1}{4}(\lambda + \tfrac{1}{2})$$

$$= (\lambda + \tfrac{1}{2})\left[\lambda(\lambda - \tfrac{1}{2}) - \tfrac{1}{4} - \tfrac{1}{4}\right]$$

$$= (\lambda + \tfrac{1}{2})\left[\lambda^2 - \tfrac{1}{2}\lambda - \tfrac{1}{2}\right]$$

$$= (\lambda + \tfrac{1}{2})(\lambda - 1)(\lambda + \tfrac{1}{2}) = 0$$

$$\lambda = -\tfrac{1}{2},\ \lambda = 1.$$

Question 2. §4.3 #~~12~~ *(5 marks)* ~~Show that in $\mathcal{P}_2$ every set with more than three vectors is linearly dependent.~~

Prove: For any vectors u, v and w in vector space V, the vectors $u - v$, $v - w$ and $w - u$ form a linearly dependent set.

$$\vec{0} = c_1(\vec{u} - \vec{v}) + c_2(\vec{v} - \vec{w}) + c_3(\vec{w} - \vec{u})$$

$$\vec{0} = (c_1 - c_3)\vec{u} + (c_2 - c_1)\vec{v} + (c_3 - c_2)\vec{w}$$

$$\begin{aligned} 0 &= c_1 - c_3 \\ 0 &= -c_1 + c_2 \\ 0 &= -c_2 + c_3 \end{aligned}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}}_{A} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The set is linearly dependent iff $|A| = 0$

$$|A| = 1 \begin{vmatrix} 1 & 0 \\ -1 & 1 \end{vmatrix} - 1 \begin{vmatrix} -1 & 1 \\ 0 & -1 \end{vmatrix}$$

$$= 1 - 1 = 0$$

$\therefore$ the set is lin. dep.