Name: Y. Lamontagne
Student ID: ______

Quiz 12

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §4.4 #13 *(5 marks)* Show that $\underbrace{\{A_1, A_2, A_3, A_4\}}_{= S}$ is a basis for $\mathcal{M}_{2\times 2}$, and express A as a linear combination of the basis vectors.

$$A_1 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, A_3 = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, A_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$$

dim $(M_{2\times 2}) = 4 = \#$ vectors in S
If S is linearly independent then S spans $M_{2\times 2}$

$$0 = c_1A_1 + c_2A_2 + c_3A_3 + c_4A_4$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = c_1\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + c_2\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} + c_3\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} + c_4\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$0 = c_1$
$0 = c_1 + c_2 \Rightarrow c_2 = 0$
$0 = c_1 + c_2 + c_3 \Rightarrow c_3 = 0$
$0 = c_1 + c_2 + c_3 + c_4 \Rightarrow c_4 = 0$
$\therefore$ only trivial solution
$\therefore$ S is linearly independent
$\therefore$ S is a basis.

$$\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} = c_1\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + c_2\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} + c_3\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} + c_4\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$c_1 = 1$
$c_1 + c_2 = 0 \Rightarrow c_2 = -1$
$c_1 + c_2 + c_3 = 1 \Rightarrow c_3 = 1$
$c_1 + c_2 + c_3 + c_4 = 0 \Rightarrow c_4 = -1$

$$\left(\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}\right)_S = (1, -1, 1, -1)$$

Question 2. §4.5 #14 *(5 marks)* Let $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ be a basis for a vector space V. Show that $\underbrace{\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}}_{S}$ is also a basis, where $\vec{u}_1 = \vec{v}_1$, $\vec{u}_2 = \vec{v}_1 + \vec{v}_2$, and $\vec{u}_3 = \vec{v}_1 + \vec{v}_2 + \vec{v}_3$.

dim $(V) = 3 = \#$ vectors in S. If S is linearly independent then S spans $M_{2\times 2}$

$0 = c_1\vec{u}_1 + c_2\vec{u}_2 + c_3\vec{u}_3$
$0 = c_1\vec{v}_1 + c_2(\vec{v}_1 + \vec{v}_2) + c_3(\vec{v}_1 + \vec{v}_2 + \vec{v}_3)$
$0 = (c_1 + c_2 + c_3)\vec{v}_1 + (c_2 + c_3)\vec{v}_2 + c_3\vec{v}_3$

since $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly independent, the only solution to the above is the trivial solution.

$0 = c_1 + c_2 + c_3 \Rightarrow c_1 = 0$
$0 = c_2 + c_3 \Rightarrow c_2 = 0$
$0 = c_3$

$\therefore$ S is linearly independent
$\therefore$ S is a Basis.