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Quiz 8

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §3.4 *(5 marks)* Find the general solution to the linear system and confirm that the row vectors of the coefficient matrix are orthogonal to the solution vectors.

$$x_1 + x_2 + x_3 = 0$$
$$2x_1 + 2x_2 + 2x_3 = 0$$
$$3x_1 + 3x_2 + 3x_3 = 0$$

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 2 & 2 & 2 & 0 \\ 3 & 3 & 3 & 0 \end{bmatrix} \sim \begin{matrix} -2R_1 + R_2 \to R_2 \\ -3R_1 + R_3 \to R_3 \end{matrix} \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Let $y = s$
$z = t$
$x = -s - t$

∴ the solution set is

$$X = (x, y, z) = (-s-t, s, t) \quad s, t \in \mathbb{R}$$

Let $\vec{a}_1 = (1, 1, 1)$
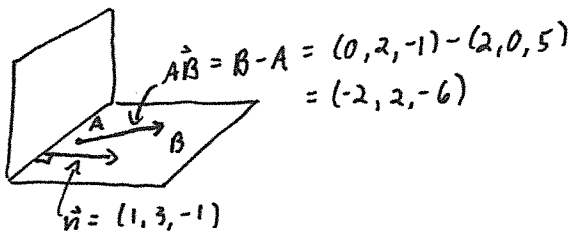$\vec{a}_2 = (2, 2, 2)$
$\vec{a}_3 = (3, 3, 3)$

$$\vec{a}_1 \cdot X = (1,1,1) \cdot (-s-t, s, t) = -s-t+s+t = 0$$
$$\vec{a}_2 \cdot X = (2,2,2) \cdot (-s-t, s, t) = -2s-2t+2s+2t = 0$$
$$\vec{a}_3 \cdot X = (3,3,3) \cdot (-s-t, s, t) = -3s-3t+3s+3t = 0$$

∴ the solution set is orthogonal to the rows of the coefficient matrix.

Question 2. 5.16 *(5 marks)* Find the equation of the plane passing through $A(2,0,5)$ and $B(0,2,-1)$ and orthogonal to the plane $x + 3y - z = 7$.

$\vec{AB} = B - A = (0, 2, -1) - (2, 0, 5)$
$= (-2, 2, -6)$

$\vec{n} = (1, 3, -1)$

Parametric equation of plane

$$(x, y, z) = (2, 0, 5) + s(1, 3, -1) + t(-2, 2, -6)$$

scalar equation of plane

$$\vec{n}_2 = \vec{n} \times \vec{AB} = \left(\begin{vmatrix} 3 & -1 \\ 2 & -6 \end{vmatrix}, -\begin{vmatrix} 1 & -1 \\ -2 & -6 \end{vmatrix}, \begin{vmatrix} 1 & 3 \\ -2 & 2 \end{vmatrix} \right)$$

$$\begin{matrix} 1 & -2 \\ 3 & 2 \\ -1 & -6 \end{matrix} = (-16, 8, 8)$$

$$-16x + 8y + 8z = d$$

Sub B
$$-16(0) + 8(2) + 8(-1) = d$$
$$8 = d$$

∴ $-16x + 8y + 8z = 8$