Dawson College: **Linear Algebra**: 201-NYC-05-S06: Fall 2015

Name: ____________________
Student ID: ____________________

Test 2

This test is graded out of 43 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Given

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 6 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 & -4 \\ -1 & 2 & 2 \\ -3 & 4 & 6 \end{bmatrix}$$

a. *(4 marks)* Find the $\text{adj}(A)$.

b. *(4 marks)* Evaluate $\det(B^{101}\text{adj}(A) + BA^{2015})$.

c. *(2 marks)* Justify that the components of the solution of the system $Ax = b$ where

$$b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

are integers.

Question 2. Given

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 9 & 3 & 4 & 1 & 0 & 9 \\ 0 & 2 & 1 & 1 & 4 & 9 & 2 & 7 & 7 & 9 \\ 0 & 0 & 1 & 1 & 1 & 4 & 9 & 2 & 7 & 7 \\ 0 & 0 & 0 & 2 & 1 & 1 & 4 & 9 & 2 & 7 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 4 & 9 & 2 \\ 0 & 0 & 0 & 0 & 0 & 2 & 1 & 1 & 4 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 & 2 & 6 \\ 2 & 3 & 3 & -3 \\ -3 & 3 & 1 & 9 \\ 0 & 1 & -1 & -12 \end{bmatrix}$$

a. *(4 marks)* Evaluate $\det(B)$.

b. *(4 marks)* If C is a square matrix and $\det(\frac{\det(B)}{2} C^T A^3) = \pi$ then find $\det(C)$, if possible.

Question 3. *(2 marks)* A non-zero square matrix A is said to be *nilpotent of degree 2* if $A^2 = 0$. Prove or disprove: There exists a square 2×2 matrix that is symmetric and nilpotent of degree 2.

Question 4. [1] Let

$$A = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & k \end{bmatrix}$$

and

$$B = \begin{bmatrix} a+2b+4c & d+2e+4f & g+2h+4k \\ 3a+4b+7c & 3d+4e+7f & 3g+4h+7k \\ 5a+7b+8c & 5d+7e+8f & 5g+7h+8k \end{bmatrix}$$

a. *(3 marks)* Find a matrix C such that $B = CA$.

b. *(2 marks)* Find the value of λ such that $\det(B) = \lambda \det(A)$ for all possible choices of A.

Question 5.[2] *(5 marks)* Suppose $\vec{u}$ and $\vec{v}$ are vectors in $\mathbb{R}^n$ such that $||\vec{u}|| = 3$, $||\vec{v}|| = 5$, and $||\vec{u}+\vec{v}|| = 7$. Find the angle between $\vec{u}$ and $\vec{v}$.

[1] From a John Abbott final examination

[2] From a John Abbott final examination

Question 6. *(5 marks)* Let $A\mathbf{x} = \mathbf{0}$ be a homogeneous system of n linear equations in n unknowns, and let Q be an invertible $n \times n$ matrix. Show that $A\mathbf{x} = \mathbf{0}$ has just the trivial solution if and only if $(QA)\mathbf{x} = \mathbf{0}$ has just the trivial solution.

Question 7. Given $\vec{u} = (1, -2, 3)$, $\vec{v} = (3, 2, 4)$, and $\vec{w} = (-4, 1, -5)$.

a. *(2 marks)* Find a unit vector that is oppositely directed to $\vec{u}$.

b. *(2 marks)* Compute $|| - ||\vec{w}||\vec{u}||$, if possible.

c. *(2 marks)* Compute $(\vec{u} \cdot \vec{v}) - \vec{w}$, if possible.

d. *(2 marks)* Sketch $\vec{v}$.

Bonus Question.*(5 marks)*
Let A and B denote invertible $n \times n$ matrices. Show that:

$$\operatorname{adj}(AB) = (\operatorname{adj}(A))(\operatorname{adj}(B))$$