Dawson College: **Linear Algebra**: 201-NYC-05-S06: Fall 2015

Name: ____________________
Student ID: ____________________

Test 3

This test is graded out of 40 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Given

$$\begin{array}{rrcll} \mathscr{A}: & (-3,-2,-1) & & & \\ \mathscr{L}_1: & (x,y,z) & = & (2+t,1-t,3t) & t\in\mathbb{R} \\ \mathscr{L}_2: & (x,y,z) & = & (2t,-3+t,2+2t) & t\in\mathbb{R} \\ \mathscr{P}_1: & x+2y+3z & = & 10 & \\ \mathscr{P}_2: & -3x-2y+z & = & 21 & \end{array}$$

a. *(3 marks)* Find an equation for the line parallel to both $\mathscr{P}_1$ and $\mathscr{P}_2$ containing $\mathscr{A}$.

b. *(4 marks)* Are $\mathscr{L}_1$ and $\mathscr{L}_2$ skew lines?

c. *(3 marks)* Are $\mathscr{L}_2$ and $\mathscr{P}_2$ parallel, perpendicular, or neither? Do they intersect? If so find the point(s) of intersection.

Question 2. *(4 marks)* Show that if $\vec{u}, \vec{v} \in \mathbb{R}^n$ then $\text{proj}_{\vec{u}}(\vec{v} - \text{proj}_{\vec{u}}(\vec{v})) = \vec{0}$. Give a geometrical interpretation of the result.

Question 3. *(4 marks)* Find all value(s) of y for which the parallelepiped generated by the vectors $(1, y, 3)$, $(2, 1, 3)$ and $(-1, -2, 1)$ has a volume of 13.

Question 4. Given the following system

$$x+z=1$$
$$y+z=1$$

a. *(2 marks)* Express a general solution of this system as a particular solution of the system plus a general solution of the associated homogeneous system.

b. *(2 marks)* Give a geometric interpretation of the result of part a. Discuss the general solution geometrically with respect to the associated homogeneous system.

Question 5.[1] *(5 marks)* Given that $\vec{u}$, $\vec{v}$, and $\vec{w}$ are three linearly independent vectors in $\mathbb{R}^n$. For which value(s) of k will the vectors $\vec{u}+2\vec{v}$, $\vec{v}+3\vec{w}$ and $k\vec{u}+\vec{w}$ be linearly dependent?

[1] From a John Abbott final examination

Question 6. Let $V = \{A \mid A \in \mathscr{M}_{2\times 2} \text{ and } \det(A) \neq 0\}$ with the following operations:

$$A + B = AB \text{ and } kA = kA$$

That is, vector addition is matrix multiplication and scalar multiplication is the regular scalar multiplication.

a. *(2 marks)* Does V satisfy closure under vector addition? Justify.

b. *(2 marks)* Does V contain a zero vector? If so find it. Justify.

c. *(2 marks)* Does V contains an additive inverse for all of its vectors? Justify.

d. *(2 marks)* Does V satisfy closure under scalar multiplication? Justify.

Question 7. *(5 marks)* Given the following two subspace of $\mathbb{R}^3$: $W_1 = \{x \mid x \in \mathbb{R}^3 \text{ and } A_1x = 0\}$ and $W_2 = \{x \mid x \in \mathbb{R}^3 \text{ and } A_2x = 0\}$ where

$$A_1 = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -3 & -3 & -3 \end{bmatrix}, A_2 = \begin{bmatrix} 5 & 7 & 9 \\ -5 & -7 & -9 \\ 10 & 14 & 18 \end{bmatrix}$$

Determine whether the two subspaces are equal or whether one of the subspaces is contained in the other. *(Hint: For each subspace determine a set of vectors that spans it.)*

Bonus Question.[2] *(5 marks)*
For n real-valued functions $f_1, \ldots, f_n$, which are $n-1$ times differentiable on an interval I, the *Wronskian* $W(f_1, \ldots, f_n)$ as a function on I is defined by

$$W(f_1, \ldots, f_n)(x) = \begin{vmatrix} f_1(x) & f_2(x) & \cdots & f_n(x) \\ f_1'(x) & f_2'(x) & \cdots & f_n'(x) \\ \vdots & \vdots & \ddots & \vdots \\ f_1^{(n-1)}(x) & f_2^{(n-1)}(x) & \cdots & f_n^{(n-1)}(x) \end{vmatrix}, \qquad x \in I.$$

Theorem: If the n real-valued functions $f_1, \ldots, f_n$, which are $n-1$ times differentiable on an interval I, and the Wronskian of these functions is not identically zero on I, then these functions form a linearly independent set of vectors on I.

Determine whether the set $\{1, e^x, \arctan x\}$ is linearly independent on $\mathbb{R}^+$

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[2]Wikipedia contributors. "Wronskian." Wikipedia, The Free Encyclopedia. Wikipedia, The Free Encyclopedia, 1 Nov. 2015. Web. 29 Nov. 2015.