

## Quiz 5

This quiz is graded out of 15 marks. No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

**Question 1.** (3 marks) Prove that if  $A$  is an invertible matrix and  $B$  is row equivalent to  $A$ , then  $B$  is also invertible.

**Question 2.** (5 marks) Let  $Ax = 0$  be a homogeneous system of  $n$  linear equations in  $n$  unknowns, and let  $Q$  be an invertible  $n \times n$  matrix. Prove that  $Ax = 0$  has only the trivial solution if and only if  $(QA)x = 0$  has only the trivial solution.

**Question 3.** (3 marks) We showed in class that the product of symmetric matrices is symmetric if and only if the matrices commute. Is the product of commuting skew-symmetric and symmetric matrices skew-symmetric? Explain.

**Question 4.** Determine whether the following statements are true or false for any  $n \times n$  matrix  $A$ . If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. (2 marks) If  $A^2$  is a symmetric matrix, then  $A$  is a symmetric matrix.

b. (2 marks) Elementary matrices are not row equivalent to the identity.