Dawson College: Fall 2019: Linear Algebra (SCIENCE): 201-NYC-05-S5: **Quiz 12** name: ______________________

No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. Let $V = \{\mathbf{x} \mid \mathbf{x} \in \mathbb{R} \text{ and } \mathbf{x} > 3\}$. For $\mathbf{u}, \mathbf{v} \in V$ and $a \in \mathbb{R}$ vector addition is defined as $\mathbf{u} \boxplus \mathbf{v} = \mathbf{uv} - 3(\mathbf{u} + \mathbf{v}) + 12$ and scalar multiplication is defined as $a \boxdot \mathbf{u} = (\mathbf{u} - 3)^a + 3$. It can be shown that $(V, \boxplus, \boxdot)$ is a vector space over the scalar field $\mathbb{R}$. Find the following:

a. *(1 mark)* $4 \boxplus 5$

b. *(1 mark)* $-2 \boxdot 4$

c. *(2 marks)* $\mathbf{0}$

d. *(2 marks)* the additive inverse of $\mathbf{x}$

Question 2. *(3 marks)* Determine whether the following is a vector space: $V = \{A | A \in \mathcal{M}_{101 \times 101} \text{ and } A \text{ is invertible}\}$ with the following operations:

$$A + B = AB \text{ and } kA = kA$$

That is, vector addition is matrix multiplication and scalar multiplication is the regular scalar multiplication. Justify.

Question 3. *(4 marks)* In any vector space V, for any $\vec{v}, \vec{w} \in V$ and $a \in \mathbb{R}$ prove that if $a\vec{v} = a\vec{w}$ and $a \neq 0$ then $\vec{v} = \vec{w}$. *Show every step, justify every step, and cite the axiom(s) used!!!*

Question 4. *(2 marks)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement. A vector space must contain at least two vectors.