Dawson College: Fall 2019: Linear Algebra (SCIENCE): 201-NYC-05-S5: **Quiz 12** name: Y. Lamontagne

No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. Let $V = \{\mathbf{x} \mid \mathbf{x} \in \mathbb{R} \text{ and } \mathbf{x} > 3\}$. For $\mathbf{u}, \mathbf{v} \in V$ and $a \in \mathbb{R}$ vector addition is defined as $\mathbf{u} \boxplus \mathbf{v} = \mathbf{uv} - 3(\mathbf{u} + \mathbf{v}) + 12$ and scalar multiplication is defined as $a \boxdot \mathbf{u} = (\mathbf{u} - 3)^a + 3$. It can be shown that $(V, \boxplus, \boxdot)$ is a vector space over the scalar field $\mathbb{R}$. Find the following:

a. *(1 mark)* $4 \boxplus 5 = 4(5) - 3(4+5) + 12 = 20 - 27 + 12 = 5$

b. *(1 mark)* $-2 \boxdot 4 = (4-3)^{-2} + 3 = 4$

c. *(2 marks)* $\mathbf{0}$

Let $\underline{0} = a$ and $\underline{u} \in V$ $\quad \underline{u} \boxplus \underline{0} = \underline{u}$

$\underline{u} \boxplus \underline{0} = \underline{u}$

$ua - 3(u+a) + 12 = u$

$ua - 3u - 3a + 12 = u$

$a(u-3) = 4u - 12$

$a = \frac{4u-12}{u-3}$

$\underline{0} = a = 4 \in V$

d. *(2 marks)* the additive inverse of $\mathbf{x}$

Since $(V, \boxplus, \boxdot)$ is a vector space we know the additive inverse of $\underline{x}$ is $-\underline{x}$ by thm 1.1.1 (note we could have used thm 1.1.1 for part c) $-\underline{x} = (x-3)^{-1} + 3$

$= \frac{1}{x-3} + 3$

Question 2.(3 marks) Determine whether the following is a vector space: $V = \{A | A \in M_{101\times101} \text{ and } A \text{ is invertible}\}$ with the following operations:

$$A + B = AB \text{ and } kA = kA$$

That is, vector addition is matrix multiplication and scalar multiplication is the regular scalar multiplication. Justify.

Not a vector space since it is not closed under scalar mult.

If $I_{101} \in V$ and $0 \in \mathbb{R}$ then $0 I_{101} = O_{101} \notin V$ since $\det(O_{101}) = 0$

therefore not invertible

Question 3. *(4 marks)* In any vector space V, for any $\vec{v}, \vec{w} \in V$ and $a \in \mathbb{R}$ prove that if $a\vec{v} = a\vec{w}$ and $a \neq 0$ then $\vec{v} = \vec{w}$. *Justify every step and cite the axiom(s) used!!!*

$a\underline{v} = a\underline{w}$

$\frac{1}{a}(a\underline{v}) = \frac{1}{a}(a\underline{w})$ since $a \neq 0$

$(\frac{1}{a}a)\underline{v} = (\frac{1}{a}a)\underline{w}$ by axiom 9.

$1\underline{v} = 1\underline{w}$

$\underline{v} = \underline{w}$ by axiom 10.

Question 4. *(2 marks)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement. A vector space must contain at least two vectors.

False

$V = \{\underline{0}\}$ with the operation $\underline{0} + \underline{0} = \underline{0}$ and $k\underline{0} = \underline{0}$ is a vector space.