Dawson College: Fall 2019: Linear Algebra (SCIENCE): 201-NYC-05-S5: **Quiz 4** name: Y. Lamontagne

No books, watches, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1.[1] *(3 marks)* Given $A = \begin{bmatrix} -2 & 4 \\ 6 & -7 \end{bmatrix}$, find a matrix X such that $XA - XA^T = A$.

$$A - A^T = \begin{bmatrix} -2 & 4 \\ 6 & -7 \end{bmatrix} - \begin{bmatrix} -2 & 6 \\ 4 & -7 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$$

$$(A-A^T)^{-1} = \frac{1}{4}\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1/2 \\ -1/2 & 0 \end{bmatrix}$$

$$X(A-A^T) = A$$
$$X(A-A^T)(A-A^T)^{-1} = A(A-A^T)^{-1}$$
$$XI = A(A-A^T)^{-1}$$
$$X = A(A-A^T)^{-1}$$
$$X = \begin{bmatrix} -2 & 4 \\ 6 & -7 \end{bmatrix}\begin{bmatrix} 0 & 1/2 \\ -1/2 & 0 \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ 7/2 & 3 \end{bmatrix}$$

Question 2.[1] *(3 marks)* Consider the matrix equation $A^{-1}B = (C - 2A)^{-1}$. Solve for A.

$$(A^{-1}B)^{-1} = ((C-2A)^{-1})^{-1}$$
$$B^{-1}A = C - 2A$$
$$B^{-1}A + 2A = C$$
$$(B^{-1}+2I)A = C$$
$$(B^{-1}+2I)^{-1}(B^{-1}+2A)A = (B^{-1}+2I)^{-1}C$$
$$IA = (B^{-1}+2I)^{-1}C$$
$$A = (B^{-1}+2I)^{-1}C$$

Question 3. Determine whether the following statements are true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. *(2 marks)* For all square matrices A and B of the same size, it is true that $(A + B)^2 = A^2 + 2AB + B^2$.

False, Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}$ $(A+B)^2 = \left(\begin{bmatrix} 4 & 2 \\ 1 & 5 \end{bmatrix}\right)^2 = \begin{bmatrix} 4 & 2 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 26 \\ 6 & 29 \end{bmatrix}$

But $A^2 + 2AB + B^2 = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} + 2\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 1 & 8 \\ 0 & 9 \end{bmatrix} + 2\begin{bmatrix} 5 & 4 \\ 3 & 6 \end{bmatrix} + \begin{bmatrix} 9 & 0 \\ 5 & 4 \end{bmatrix} = \begin{bmatrix} 20 & 16 \\ 11 & 25 \end{bmatrix}$.

b. *(2 marks)* The sum of two invertible matrices of the same size must be invertible.

False, $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ are both invertible since $ad - bc \neq 0$

But $A + B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is not invertible since $ad - bc = 0$

Question 4. *(3 marks)* Prove: If A has two rows which are identical then A is singular.

Suppose the i^{th} and j^{th} row of A are identical. If the exist a matrix B such that i^{th} row of the product AB is $[0 \dots 0\, 1\, 0 \dots 0]$ then j^{th} row of the product AB is the same.

$\uparrow i^{th}$ column

Hence it is impossible to find a matrix B s.t. $AB = I$.

$\therefore$ A is singular.

Bonus. *(3 marks)* Find the formula for the n-th power of this matrix. $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

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[1]From a past John Abbott final examination