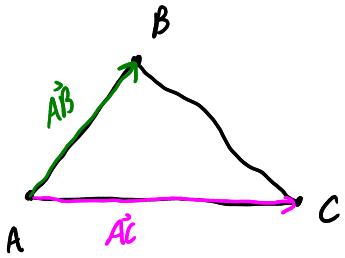
Dawson College: **Linear Algebra**: 201-105-DW-S5: Fall 2022: **Quiz 11** name: _____

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. Given $A(1,2,3), B(0,1,-2)$ and $C(-1,0,5)$

a. *(4 marks)* Find the area of the triangle ABC.



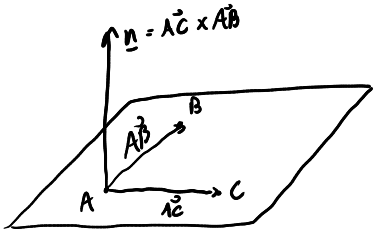
$$\text{Area} = \frac{\|\vec{AC} \times \vec{AB}\|}{2}$$

$$\vec{AC} = \underline{C} - \underline{A} = (-1,0,5) - (1,2,3) = (-2,-2,2)$$
$$\vec{AB} = \underline{B} - \underline{A} = (0,1,-2) - (1,2,3) = (-1,-1,-5)$$

$$\vec{AC} \times \vec{AB} = \begin{pmatrix} \begin{vmatrix} -2 & -1 \\ -2 & -5 \end{vmatrix}, -\begin{vmatrix} -2 & -1 \\ 2 & -5 \end{vmatrix}, \begin{vmatrix} -2 & -1 \\ 2 & -1 \end{vmatrix} \end{pmatrix}$$
$$= (12, -12, 0)$$

$$\text{Area} = \frac{\|(12,-12,0)\|}{2} = \frac{\sqrt{12^2 + (-12)^2 + 0^2}}{2} = \frac{\sqrt{288}}{2}$$
$$= \sqrt{72}$$
$$= 6\sqrt{2}$$

b. *(3 marks)* Find the general and parametric equation of the plane that contains the points A, B and C.



parametric eqn: $\underline{x} = A + s\vec{AC} + t\vec{AB} \quad s,t \in \mathbb{R}$
$= (1,2,3) + s(-2,-2,2) + t(-1,-1,-5)$

general eqn: $ax + by + cz = d$
$12x - 12y = d$

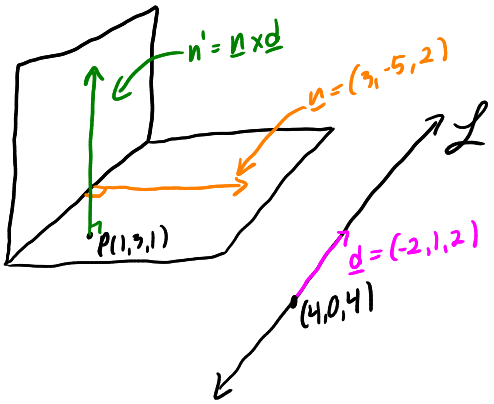
sub in A to solve for d
$12(1) - 12(2) = d$
$-12 = d$

$\therefore 12x - 12y = -12.$

Question 2. *(4 marks)* Simplify $(\vec{u}+\vec{v}) \times (\vec{u}-\vec{v})$ and write as a single term.

$= \underline{u} \times \underline{u} + \underline{u} \times (-\underline{v}) + \underline{v} \times \underline{u} + \underline{v} \times (-\underline{v})$
$= \underline{0} - \underline{u} \times \underline{v} + \underline{v} \times \underline{u} - \underline{v} \times \underline{v}$
$= \underline{v} \times \underline{u} + \underline{v} \times \underline{u} - \underline{0}$
$= 2(\underline{v} \times \underline{u})$

Questions 3. *(5 marks)* Find the equation of the plane through the point $P(1,3,1)$ that is parallel to the line $(x,y,z) = (4,0,4) + t(-2,1,2) \quad t \in \mathbb{R}$ and perpendicular to the plane $3x - 5y + 2z = 13$.



$\underline{n}' = \underline{n} \times \underline{d} = \left(\begin{vmatrix} -5 & 1 \\ 2 & 2 \end{vmatrix}, -\begin{vmatrix} 3 & -2 \\ 2 & 2 \end{vmatrix}, \begin{vmatrix} 3 & -2 \\ -5 & 1 \end{vmatrix}\right)$

$\begin{matrix} 3 & -2 \\ -5 & 1 \\ 2 & 2 \end{matrix} = (-12, -10, -7)$

$ax + by + cz = d$

$-12x - 10y - 7z = d$

sub P to solve for d

$-12(1) - 10(3) - 7(1) = d$

$-49 = d$

$\therefore -12x - 10y - 7z = -49$