

Question 1. (2 marks) Determine whether the equations form a linear system.

$$\begin{aligned} 3xy + x &= -4 \\ y + 5z &= 1 \\ 6x + 2z &= 3 \\ -x - y - z &= 4 \end{aligned}$$

Not a system of linear equations since the first equation is not linear. It is not linear since two variables are multiplied.

Question 2. (3 marks) Find all values of k for which the given augmented matrix corresponds to a consistent linear system.

$$\begin{bmatrix} 3 & -4 & k \\ -6 & 8 & 5 \end{bmatrix}$$

$$\begin{cases} 3x - 4y = k \\ -6x + 8y = 5 \end{cases}$$

$$\begin{cases} y = \frac{3}{4}x - \frac{k}{4} \\ y = \frac{3}{4}x + \frac{5}{8} \end{cases}$$

\therefore both lines are parallel since their slopes are equal. For the system to be consistent the y-intercept must be equal $\frac{-k}{4} = \frac{5}{8}$
 $k = -\frac{5}{2}$

Question 3. (3 marks) Determine whether the given vector $(3, 1, 1)$ is a solution of the linear system

$$\begin{aligned} 2x_1 - 4x_2 - x_3 &= 1 \\ x_1 - 3x_2 + x_3 &= 1 \\ 3x_1 - 5x_2 - 3x_3 &= 1 \end{aligned}$$

Let's verify whether the solution satisfies each equation

$$\begin{aligned} 2(3) - 4(1) - 1 &= 1 \checkmark \\ 3 - 3(1) + 1 &= 1 \checkmark \\ 3(3) - 5(1) - 3(1) &= 1 \checkmark \end{aligned}$$

$\therefore (3, 1, 1)$ is a solution

Question 4. (4 marks) Find the solution set of the linear equation by using parameters as necessary

$$3x_1 - 5x_2 + 4x_3 = 7$$

Also find two particular solutions.

$$\text{Let } \begin{aligned} x_2 &= s \\ x_3 &= t \end{aligned} \quad s, t \in \mathbb{R}$$

sub into eqn

$$\begin{aligned} 3x_1 - 5s + 4t &= 7 \\ 3x_1 &= 7 + 5s - 4t \\ x_1 &= \frac{7}{3} + \frac{5}{3}s - \frac{4}{3}t \end{aligned}$$

$$\therefore x_1 = \frac{7}{3} + \frac{5}{3}s - \frac{4}{3}t$$

$$x_2 = s \quad s, t \in \mathbb{R}$$

Particular sol.

$$1) \quad s = t = 0$$

$$\therefore (x_1, x_2, x_3) = \left(\frac{7}{3}, 0, 0\right)$$

$$2) \quad s = 0, t = 1$$

$$\therefore (x_1, x_2, x_3) = (1, 0, 1)$$