**Dawson College**: **Linear Algebra** (SCIENCE): 201-NYC-05-S7: Fall 2022: **Quiz 5** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

**Question 1.**[1] *(5 marks)* Consider the following Gauss-Jordan reduction:

$$\underbrace{\begin{bmatrix} -2 & 5 & 4 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{A} \underset{-4R_3+R_1\to R_1}{\sim} \underbrace{\begin{bmatrix} -2 & 5 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{E_1A} \underset{R_1\leftrightarrow R_2}{\sim} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{E_2E_1A} \underset{2R_1+R_2\to R_2}{\sim} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{E_3E_2E_1A} \underset{\frac{1}{5}R_2\to R_2}{\sim} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{E_4E_3E_2E_1A} = I$$

Find $E_1, E_2, E_3, E_4$ and express $A$ as a product of elementary matrices.

$E_1$: $I_3 \sim -4R_3+R_1\to R_1 \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = E_1$

$E_2$: $I_3 \sim R_1 \leftrightarrow R_2 \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = E_2$

$E_3$: $I_3 \sim 2R_1+R_2\to R_2 \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = E_3$

$E_4$: $I_3 \sim \frac{1}{5}R_2 \to R_2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/5 & 0 \\ 0 & 0 & 1 \end{bmatrix} = E_4$

$E_4E_3E_2E_1A = I$

$A$ is invertible since its RREF is $I$

$E_4E_3E_2E_1AA^{-1} = IA^{-1}$

$E_4E_3E_2E_1 = A^{-1}$

$(E_4E_3E_2E_1)^{-1} = (A^{-1})^{-1}$

$A = E_1^{-1}E_2^{-1}E_3^{-1}E_4^{-1}$

$E_1^{-1}$: $I_3 \sim 4R_3+R_1\to R_1 \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = E_1^{-1}$

$E_2^{-1}$: $I_3 \sim R_1\leftrightarrow R_2$ $E_2$

$E_3^{-1}$: $I_3 \sim -2R_1+R_2\to R_2 \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = E_3^{-1}$

$E_4^{-1}$: $I_3 \sim 5R_2\to R_2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix} = E_4^{-1}$

**Question 2.** Determine whether the following statements are true or false for any $n \times n$ matrices $A$ and $B$. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

1. *(3 marks)* If $A$ is an invertible matrix and $B$ is row equivalent to $A$, then $B$ is also invertible.

True, since $A$ is invertible it can be expressed as a product of elementary matrices by the equivalence thm. $A = E_1E_2\cdots E_k$. Since $B$ is row equivalent to $A$ there exists elementary matrices $F_i$ s.t. $F_\ell \cdots F_2F_1A = B$

Which implies that $B = F_\ell \cdots F_2F_1E_1E_2\cdots E_k$

$\therefore$ $B$ is invertible since it can be expressed as a product of elementary matrices.

2. *(3 marks)* An expression of an invertible matrix $A$ as a product of elementary matrices is unique.

False, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ can be expressed as the following two product of elementary matrices

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ where $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is elementary since

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \sim 1R_1 \to R_1 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = E$

[1] from WeBWorK