

Question 1. (5 marks) Determine conditions on the b_i 's, if any, in order to guarantee that the linear system is consistent. And discuss your result using the Equivalence Theorem.

$$\begin{cases} x_1 - 2x_2 - x_3 = b_1 \\ -4x_1 + 5x_2 + 2x_3 = b_2 \\ -4x_1 + 7x_2 + 4x_3 = b_3 \end{cases}$$

Questions 2. (1 mark) Create a skew-symmetric matrix (that is, $A^T = -A$) by substituting appropriate numbers for the x 's.

$$\begin{bmatrix} 0 & x & x & x \\ 3 & 0 & x & x \\ 7 & -8 & 0 & x \\ 2 & -3 & 9 & 0 \end{bmatrix}$$

Question 3. (4 marks) Prove: If $A^T A = A$, then A is symmetric and $A = A^2$.