Dawson College: **Linear Algebra** (COMPUTER SCIENCE): 201-NYC-05-S9: Fall 2022: **Quiz 8** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. Given A and B are 3×3 matrices, $\det(A)=4$ and $\det(B)=-3$. Find

a. *(4 marks)* $\det(-A^T(2B)^{-3}) = (-1)^3 \det(A^T(2B)^{-3})$

$= -\det(A^T)\det((2B)^{-3})$

$= -\det(A)\dfrac{1}{\det((2B)^3)}$

$= -\det(A)\dfrac{1}{(\det(2B))^3}$

$= -\det(A)\dfrac{1}{(2^3\det(B))^3} = -4\dfrac{1}{(2^3(-3))^3} = \dfrac{-4}{2^9(-3)^3} = \dfrac{1}{2^7 3^3}$

b. *(4 marks)* $\det(5BA^{-1}-2B\operatorname{adj}(A)) = \det(BA^{-1}-2B(\det(A)A^{-1})$

$A^{-1} = \dfrac{1}{\det(A)}\operatorname{adj}(A)$

$\operatorname{adj}(A) = (\det(A))A^{-1}$

$= \det(BA^{-1}-2(\det(A))BA^{-1})$

$= \det(BA^{-1}-2(4)BA^{-1})$

$= \det(-7BA^{-1})$

$= (-7)^3\det(BA^{-1})$

$= (-7)^3\det(B)\det(A^{-1})$

$= (-7)^3(-3)\dfrac{1}{\det A}$

$= (-7)^3(-3)\dfrac{1}{4} = \dfrac{(-7)^3(-3)}{4}$

Question 2. Given the linear system $\begin{cases} -2x_1+3x_2+2x_3=1 \\ x_1-\ x_2+4x_3=2 \\ -3x_1+2x_2+\ x_3=5 \end{cases}$

a. *(3 marks)* Find the first column of the adjoint of the coefficient matrix of the above system.

$\operatorname{adj} A = \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}$

$C_{11} = (-1)^{1+1}\begin{vmatrix} -1 & 4 \\ 2 & 1 \end{vmatrix} = -9$

$C_{12} = (-1)^{1+2}\begin{vmatrix} 1 & 4 \\ -3 & 1 \end{vmatrix} = -(1+12) = -13$

$= \begin{bmatrix} -9 & C_{21} & C_{31} \\ -13 & C_{22} & C_{32} \\ -1 & C_{23} & C_{33} \end{bmatrix}$

$C_{13} = (-1)^{1+3}\begin{vmatrix} 1 & -1 \\ -3 & 2 \end{vmatrix} = 2-3 = -1$

b. *(4 marks)* Find x_2 of the above system only by using Cramer's Rule.

$|A| = \begin{vmatrix} -2 & 3 & 2 \\ 1 & -1 & 4 \\ -3 & 2 & 1 \end{vmatrix} = a_{11}C_{11}+a_{12}C_{12}+a_{13}C_{13} = -2\begin{vmatrix} -1 & 4 \\ 2 & 1 \end{vmatrix} - 3\begin{vmatrix} 1 & 4 \\ -3 & 1 \end{vmatrix} + 2\begin{vmatrix} 1 & -1 \\ -3 & 2 \end{vmatrix} = -2(-9)-3(13)+2(-1)$
$= -23$

$|A_2| = \begin{vmatrix} -2 & 1 & 2 \\ 1 & 2 & 4 \\ -3 & 5 & 1 \end{vmatrix} = -2\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} - \begin{vmatrix} 1 & 4 \\ -3 & 1 \end{vmatrix} + 2\begin{vmatrix} 1 & 2 \\ -3 & 5 \end{vmatrix} = -2(2-20)-(1+12)+2(5+6)$
$= -2(-18)-13+2(11)$
$= 45$

$\therefore x_2 = \dfrac{|A_2|}{|A|} = \dfrac{45}{-23}$