Dawson College: **Linear Algebra** (COMPUTER SCIENCE): 201-NYC-05-S9: Fall 2022: **Quiz 11** name: Y. Lamontagne

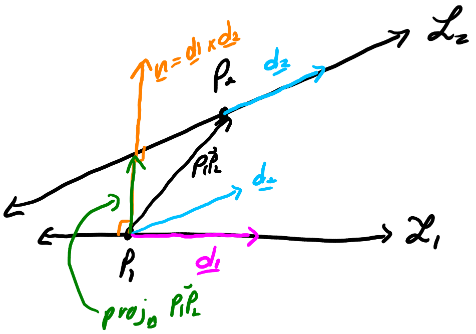
Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* Given the non-intersecting lines:

$$\begin{array}{lcl} \mathscr{L}_1 & : & (x,y,z) = \overbrace{(1,2,-2)}^{P_1} + t_1\overbrace{(1,2,1)}^{d_1} \\ \mathscr{L}_2 & : & (x,y,z) = \underbrace{(2,1,3)}_{P_2} + t_2\underbrace{(1,2,3)}_{d_2} \text{ where } t_1,\ t_2 \in \mathbb{R}. \end{array}$$

Find the shortest distance between $\mathscr{L}_1$ and $\mathscr{L}_2$.

Since $\underline{d}_1 \nparallel \underline{d}_2$ and $\mathscr{L}_1$ and $\mathscr{L}_2$ are non-intersecting then $\mathscr{L}_1$ and $\mathscr{L}_2$ are skew-lines.



$$\vec{P_1P_2} = P_2 - P_1$$
$$= (2,1,3)-(1,2,-2)$$
$$= (1,-1,5)$$

$$\underline{n} = \underline{d}_1 \times \underline{d}_2 = \left(\begin{vmatrix} 2 & 2 \\ 1 & 3 \end{vmatrix}, -\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} \right) = (4,-2,0)$$

$$\begin{array}{cc} 1 & 1 \\ 2 & 2 \\ 1 & 3 \end{array}$$

$$proj_{\underline{n}} \vec{P_1P_2} = \frac{\underline{n} \cdot \vec{P_1P_2}}{\underline{n} \cdot \underline{n}} \underline{n}$$
$$= \frac{(4,-2,0)\cdot(1,-1,5)}{(4,-2,0)\cdot(4,-2,0)}(4,-2,0)$$
$$= \frac{4(1)+(-2)(-1)+0(5)}{4(4)+(-2)(-2)+0(0)}(4,-2,0)$$
$$= \frac{6}{20}(4,-2,0)$$
$$= \frac{3}{10}(4,-2,0)$$
$$= \frac{3}{5}(2,-1,0)$$

$$\text{distance} = \| proj_{\underline{n}} \vec{P_1P_2} \|$$
$$= \left\| \frac{3}{5}(2,-1,0) \right\| = \frac{3}{5}\|(2,-1,0)\| = \frac{3}{5}\sqrt{2^2+(-1)^2+0^2}$$
$$= \frac{3}{5}\sqrt{5}$$

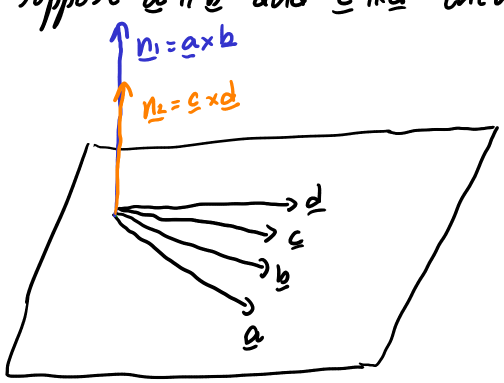
Questions 2. *(4 marks)* Prove: If $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ lie in the same plane, then $\underbrace{(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})}_{S} = \vec{0}$.

If $\underline{a} \parallel \underline{b}$ then $\exists k$ s.t. $\underline{b} = k\underline{a}$ $\Rightarrow$ $\underline{a}\times\underline{b} = \underline{a}\times(k\underline{a}) = k(\underline{a}\times\underline{a}) = k\underline{0} = \underline{0}$

$\Rightarrow S = \underline{0}$

Similarly if $\underline{c} \parallel \underline{d}$ $\Rightarrow S = \underline{0}$

Suppose $\underline{a} \nparallel \underline{b}$ and $\underline{c} \nparallel \underline{d}$ then

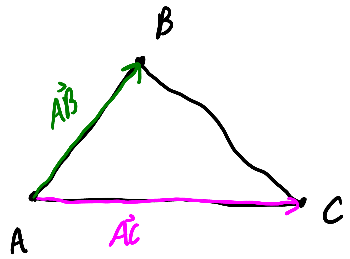


Since $\underline{n}_1 \parallel \underline{n}_2$ $\exists k$ s.t. $\underline{n}_2 = k\underline{n}_1$

$$S = \underline{n}_1 \times \underline{n}_2$$
$$= \underline{n}_1 \times (k\underline{n}_1)$$
$$= k(\underline{n}_1 \times \underline{n}_1)$$
$$= k\underline{0}$$
$$= \underline{0}$$

Questions 3. Given $A(1,2,3), B(0,1,-2)$ and $C(-1,0,5)$

a. *(4 marks)* Find the area of the triangle ABC.



$$\text{Area} = \frac{\|\vec{AC} \times \vec{AB}\|}{2}$$

$$\vec{AC} = \underline{C} - \underline{A} = (-1,0,5) - (1,2,3) = (-2,-2,2)$$

$$\vec{AB} = \underline{B} - \underline{A} = (0,1,-2) - (1,2,3) = (-1,-1,-5)$$

$$\vec{AC} \times \vec{AB} = \left(\begin{vmatrix} -2 & -1 \\ 2 & -5 \end{vmatrix}, -\begin{vmatrix} -2 & -1 \\ 2 & -5 \end{vmatrix}, \begin{vmatrix} -2 & -1 \\ -2 & -1 \end{vmatrix} \right)$$

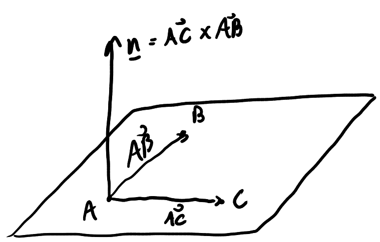
$$\begin{matrix} -2 & -1 \\ -2 & -1 \\ 2 & -5 \end{matrix} \quad = (12, -12, 0)$$

$$\text{Area} = \frac{\|(12,-12,0)\|}{2} = \frac{\sqrt{12^2 + (-12)^2 + 0^2}}{2} = \frac{\sqrt{288}}{2}$$

$$= \sqrt{72}$$

$$= 6\sqrt{2}$$

b. *(3 marks)* Find the general and parametric equation of the plane that contains the points A, B and C.



parametric eqn: $\underline{x} = A + s\vec{AC} + t\vec{AB} \quad s,t \in \mathbb{R}$

$$= (1,2,3) + s(-2,-2,2) + t(-1,-1,-5)$$

general eqn: $ax + by + cz = d$

$$12x - 12y = d$$

sub in A to solve for d

$$12(1) - 12(2) = d$$

$$-12 = d$$

$\therefore 12x - 12y = -12.$