DAWSON COLLEGE
MATHEMATICS DEPARTMENT
MATHEMATICS NYB - REGULAR
MAY 17, 2005

Marks

1) Find the indefinite integrals.

i) $\int x\sqrt{x-1}\,dx$

ii) $\int \frac{1}{x^2+4x+5}\,dx$

iii) $\int \sqrt{x}\;\ell n\; x\,dx$

iv) $\int \sin^2 x \cos^3 x\,dx$

v) $\int \frac{5x^2+12}{x\left(x^2+4\right)}\,dx$

vi) $\int \frac{x^2}{\sqrt{9-x^2}}\,dx$ /30

2) Use the definition of the definite integral to evaluate $\int_{x=0}^{2} \left(x^2-3\right)dx$.

NOTE: $\sum_{k=1}^{n} 1 = n$ $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

/10

3) Find the area of the region bounded by $f(x) = -x^2+3$ and $g(x) = 2x$. /5

4) Find the volume of the solid which results by rotating the region bounded by $f(x) = -x^2 + 4x$ and $g(x) = x^2$ about the y-axis. /5

5) Find the volume of the region which results by rotating $f(x) = 3x + 1$ and $g(x) = x^2 + 1$ about the x-axis. /5

6) Find the limits if they exist.

i) $\lim_{x \to \infty} x \sin\left(\frac{1}{x}\right)$

ii) $\lim_{x \to \infty} \left(1 - \frac{2}{x}\right)^{3x}$ /8

7) For each integral state whether the integral converges or diverges. If the integral converges state the number it converges to.

i) $\int_{x=0}^{2} \frac{1}{\sqrt{2-x}} dx$

ii) $\int_{x=0}^{\infty} e^{-3x} dx$ /10

8) Find the length of the arc of the graph of $f(x) = \frac{x^4}{8} + \frac{1}{4x^2}$ from $x = 1$ to $x = 2$. /6

9) Evaluate $\sum_{n=1}^{\infty} \frac{2^n}{3^{n+2}}$. /4

10) Explain why each of the following series converges or diverges citing an appropriate test in each case.

i) $\sum_{n=3}^{\infty} \frac{1}{n(\ell n\, n)^2}$

ii) $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^2 - n}}$

iii) $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$ /12

11) Write the 4th degree Maclaurin Polynomial $P_4(x)$ of $f(x) = \cos(3x)$. /5