Dawson College
Department of Mathematics

Final Examination (Calculus II 201-NYB-05 Science)

Winter 2007
Instructors: Beck, Chaubey, Dubrovsky, Fournier, Fox, Frajberg, Zlotchevskaia
Time: 3 hours

Name: ______________________

ID #: ANSWERS

#	Mark
1. a)	
b)	
c)	
d)	
2. a)	
b)	
c)	
d)	
3.	
4.	
5.	
6.	
7.	
8	
9.	
10.	
11.	
12.	
13.	
14.	

Note: Scientific, non-programmable calculators are permitted.

This exam consists of 20 questions, 5 marks/question.

1. Compute the antiderivatives.

a) $\int x\ln(x)dx.$ $\frac{1}{2}x^2\ln(x) - x^2/4 + C$

b) $\int \frac{1}{x^2+x+1}dx.$ $\frac{2}{\sqrt{3}}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)+C$

c) $\int \frac{x+1}{\sqrt{x^2+2x}}dx.$ $(x^2+2x)^{1/2}+C$

d) $\int \frac{1}{x(x+1)^2}dx.$ $\int\left(\frac{1}{x}-\frac{1}{x+1}-\frac{1}{(x+1)^2}\right)dx = \ln|x|-\ln|x+1|+\frac{1}{x+1}+C$

2. Compute the definite integrals.

a) $\int_0^{\pi/4} \tan(x)\sec^4(x)\,dx.$ $\left.\frac{\tan^4(x)}{4}+\frac{\tan^2(x)}{2}\right]_0^{\pi/4} = \frac{3}{4}$

b) $\int_2^3 \frac{x}{\sqrt{x^2-1}}dx.$ $\left.\sqrt{x^2-1}\right]_2^3 = \sqrt{8}-\sqrt{3} = 2\sqrt{2}-\sqrt{3}$

c) $\int_{-\pi}^{\pi} x\sin(x^2)\,dx.$ $\left.\frac{1}{2}\sin(x^2)\right]_{-\pi}^{\pi} = 0$

d) $\int_0^1 \frac{1}{1+e^x}dx.$ $\left.x-\ln(1+e^x)\right]_0^1 = 1-\ln(1+e)+\ln(2)$

3. Evaluate the definite integral $\int_0^1 x^3\,dx$ by using the limit definition.

Hint: $\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}.$

$\frac{1}{n}\sum_{k=1}^{n}\frac{k^3}{n^3} = \frac{1}{n^4}\frac{(n+1)^2n^2}{4} = \left(1+\frac{1}{n}\right)^2\frac{1}{4} \Rightarrow \frac{1}{4}$

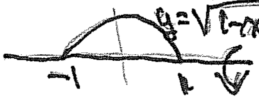
4. Compute the derivative: $\frac{d}{dx}\int_0^{2x}\cos\left(-t^2\right)dt.$ $2\cos(-4x^2)$

5. Find the area of the plane region bounded by the curves

$x=0,\ y=\sin(x),\ y=\cos(x),\ x=\pi/4.$

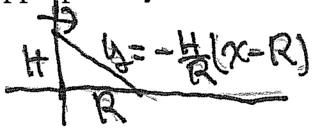
$$\int_0^{\pi/4} (\cos(x)-\sin(x))\,dx = \sin(x)+\cos(x)\Big]_0^{\pi/4} = \sqrt{2}-1$$

6. Find the volume of the sphere of radius 1 by rotating the area under the graph of the half-circle $x^2+y^2=1 \quad (y \geq 0)$ about the x-axis.



$$\pi\int_{-1}^{1} \left(\sqrt{1-x^2}\right)^2 dx = 2\pi\left(x-x^3/3\right)\Big]_0^1 = \frac{4\pi}{3}$$

7. Find the volume of a circular cone with height H and base radius R by rotating an appropriately chosen triangle about the y-axis.



$$2\pi\int_0^R -x\frac{H}{R}(x-R)\,dx = -\frac{2\pi H}{R}\left[\frac{x^3}{3}-\frac{Rx^2}{2}\right]_0^R = \pi R^2 H/3$$

8. Find the length of the graph of $y=\dfrac{e^x+e^{-x}}{2}$ over the interval $[0,1]$.

$$\int_0^1 \sqrt{1+\left(\frac{e^x-e^{-x}}{2}\right)^2}\,dx = \int_0^1 \frac{e^x+e^{-x}}{2}\,dx = \frac{e^x-e^{-x}}{2}\Big]_0^1 = \frac{e-1/e}{2}$$

9. Compute: $\displaystyle\lim_{x\to0^+}\frac{\sqrt{x+1}-1}{\sqrt{x}}$.

$$\frac{0}{0};\ \lim_{x\to0^+}\frac{\frac{2}{\sqrt{x+1}}}{\frac{2}{\sqrt{x}}} = \lim_{x\to0^+}\frac{\sqrt{x}}{\sqrt{x+1}} = 0$$

10. Compute: $\displaystyle\lim_{x\to\infty} x^2e^{-x^2}$.

$$\infty\cdot 0;\ \lim_{x\to\infty}\frac{x^2}{e^{x^2}} = \lim_{x\to\infty}\frac{2x}{2xe^{x^2}} = \lim_{x\to\infty}\frac{1}{e^{x^2}} = 0$$

11. Compute: $\displaystyle\int_0^\infty xe^{-x}\,dx$.

$$\lim_{M\to\infty} -xe^{-x}-e^{-x}\Big]_0^M = \lim_{M\to\infty} -Me^{-M}-e^{-M}+1 = 1$$

12. Compute: $\displaystyle\sum_{n=0}^{\infty}\frac{2^n}{3^{n+1}}$.

$$\frac{1}{3}\times\frac{1}{1-2/3} = 1$$

13. Test for convergence: $\displaystyle\sum_{n=4}^{\infty}\frac{2}{n(n-2)}$.

$$= \sum_{n=4}^{\infty}\frac{1}{n-2}-\frac{1}{n} = \sum_{n=4}^{\infty}\frac{1}{n-2}-\frac{1}{n-1} + \sum_{n=4}^{\infty}\frac{1}{n-1}-\frac{1}{n}$$

CONV $\quad = 1/2 + 1/3$

14. Test for convergence: $\displaystyle\sum_{n=0}^{\infty}\frac{(n!)^2}{(2n)!}$.

$$\lim_{n\to\infty}\frac{\frac{((n+1)!)^2}{(2n+2)!}}{\frac{(n!)^2}{(2n)!}} = \lim_{n\to\infty}\frac{(n+1)^2}{(2n+2)(2n+1)} = \lim_{n\to\infty}\frac{n+1}{2(2n+1)} = \frac{1}{4} < 1$$

CONV