DAWSON COLLEGE – WINTER 2007 – MATHEMATICS DEPARTMENT

FINAL EXAMINATION
201-NYB-05 CALCULUS II-REGULAR

1. Find the following integrals:

(5 points) a) $\int \frac{2x}{\sqrt[5]{x-4}}\,dx$

(5 points) b) $\int \frac{\ln\left(\sqrt{x}+3\right)}{\sqrt{x}}\,dx$

(5 points) c) $\int \sin^3\frac{\pi x}{7}\cos^2\frac{\pi x}{7}\,dx$

(5 points) d) $\int x^2 e^{3x}\,dx$

(5 points) e) $\int \frac{-4x^3}{\sqrt{x^2+16}}\,dx$

(5 points) f) $\int_1^5 \frac{2x-2}{x^2(x+1)}\,dx$

2. (5 points) Use the definition of the definite integral to evaluate:

$$\int_0^5 \left(-x^2+5x\right)dx$$

Note: $\sum_{i=1}^{n} 1 = n, \sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

3. Consider the region $\mathcal{R}$ bounded by : $y = 3x$ **and** $y = -x^2 + 4x$ **.**

(a) (5 points) Find the area of the region $\mathcal{R}$.

(b) (5 points) Find the volume of the solid obtained by rotating the region $\mathcal{R}$ around the x - axis. Illustrate with a figure.

(c) (5 points) Find the volume of the solid obtained by rotating the region $\mathcal{R}$ around the y - axis. Illustrate with a figure.

4. (5 points) Find the length of the function $f(x) = \frac{2}{3}(2x+1)^{\frac{3}{2}}$ **from** $x = 0$ **to** $x = 1$**.**

5. (5 points) Find the average value of the function $f(x)=\dfrac{3}{x+2}$ **from** $x=1$ **to** $x=2$**.**

6. Evaluate each limit if it is possible:

(a) (4 points) $\lim\limits_{x\to 0^+}\dfrac{e^x-x-1}{3x^2}$

(b) (4 points) $\lim\limits_{x\to 1^+}(3x-3)^{x-1}$

7. For each improper integral state whether the integral converges or diverges. If the integral converges state a number it converges to:

(a) (5 points) $\int_4^5 \dfrac{1}{(x-4)^{\frac{2}{3}}}\,dx$

(b) (5 points) $\int_0^\infty \dfrac{e^x}{4+e^x}\,dx$

8. Study the following series. Tell if they are convergent or divergent.(Indicate the test that you use in each case):

(a) (4 points) $\sum\limits_{n=1}^{\infty}\dfrac{n^2 5^{n+1}}{7^n}$

(b) (4 points) $\sum\limits_{n=1}^{\infty}\dfrac{2}{(n+2)(n+3)}$

(c) (4 points) $\sum\limits_{n=3}^{\infty}\dfrac{\ln n}{n}$

9. (5 points) Find the exact value of $\sum\limits_{n=0}^{\infty}\dfrac{2^n+3^n}{7^n}$**.**

10. (5 points) Write the 4-th degree Maclaurin polynomial $T_4(x)$ **for** $f(x)=-3\cos(2x)$**.**

Answers:

1. (a) $\frac{10}{9}(x-4)^{\frac{9}{5}}+10(x-4)^{\frac{4}{5}}+C$

(b) $2\left(\sqrt{x}+3\right)\ln\left(\sqrt{x}+3\right)-2\sqrt{x}-6+C$

(c) $-\frac{7}{\pi}\left(\frac{\cos^3\left(\frac{\pi x}{7}\right)}{3}-\frac{\cos^5\left(\frac{\pi x}{7}\right)}{5}\right)+C$

(d) $\frac{x^2}{3}e^{3x}-\frac{2x}{9}e^{3x}+\frac{2}{27}e^{3x}+C$

(e) $-256\left(\frac{\left(\frac{\sqrt{x^2+16}}{4}\right)^3}{3}-\frac{\sqrt{x^2+16}}{4}\right)+C$

(f) $4\ln\frac{5}{3}-\frac{8}{5}$

2. $\frac{125}{6}$

3. (a) $A=\frac{1}{6}u^2$ **(b)** $V=\frac{8\pi}{15}u^3$ **(c)** $V=\frac{\pi}{6}u^3$

4. $\frac{1}{12}\left(13^{\frac{3}{2}}-5^{\frac{3}{2}}\right)$

5. $3\ln\frac{4}{3}$

6. (a) $\frac{1}{6}$ **(b)** 1

7. (a) 3 **(b)** divergent

8. (a) convergent **(b)** convergent **(c)** divergent

9. $\frac{63}{20}$

10. $-2x^4+6x^2-3$