

Quiz 3

This quiz is graded out of 10 marks. No books, graphing calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (3 marks) Integrate the following indefinite integral:

$$\int x \csc x \cot x \, dx = uv - \int v \, du$$

$$u = x \quad du = dx$$

$$v = -\csc x \quad dv = \csc x \cot x \, dx$$

$$= -x \csc x - \int -\csc x \, dx$$

$$= -x \csc x - \ln |\csc x + \cot x| + C$$

Question 2. (3 marks) Integrate the following indefinite integral:

$$\int \cos^2 x \sin^2 x \, dx = \int \left(\frac{1 + \cos 2x}{2} \right) \left(\frac{1 - \cos 2x}{2} \right) dx$$

$$= \frac{1}{4} \int 1 - \cos^2 2x \, dx$$

$$= \frac{1}{4} \int 1 - \left[\frac{1 + \cos 4x}{2} \right] dx$$

$$= \frac{1}{4} \int \frac{1}{2} + \frac{\cos 4x}{2} \, dx$$

$$= \frac{1}{4} \left[\frac{x}{2} + \frac{\sin 4x}{8} \right] + C$$

$$= \frac{x}{8} + \frac{\sin 4x}{32} + C$$

Question 3. (4 marks) Integrate the following indefinite integral:

$$\int \frac{\sqrt{x^2-4}}{x} dx$$

$$x = 2\sec\theta$$

$$dx = 2\sec\theta \tan\theta d\theta$$

$$= \int \frac{\sqrt{(2\sec\theta)^2-4}}{2\sec\theta} 2\sec\theta \tan\theta d\theta$$

$$= \int \sqrt{4\tan^2\theta} \tan\theta d\theta$$

$$= \int 2\tan^2\theta d\theta$$

$$= 2 \int \sec^2\theta - 1 d\theta$$

$$= 2 \left[\tan\theta - \theta \right] + C$$

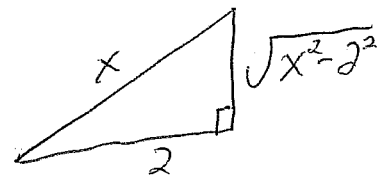
$$= 2 \left[\frac{\sqrt{x^2-4}}{2} - \operatorname{arcsec} \frac{x}{2} \right] + C$$

$$x = 2\sec\theta$$

$$\frac{x}{2} = \sec\theta$$

$$\theta = \operatorname{arcsec} \frac{x}{2}$$

$$\frac{h}{a} = \frac{x}{2} = \sec\theta$$



$$\tan\theta = \frac{h}{a} = \frac{\sqrt{x^2-4}}{2}$$