Dawson College: **Calculus II**: 201-NYB-05-S1: Summer 2008

Name: ____________________
Student ID: ____________________

Quiz 4

This quiz is graded out of 12 marks. No books, graphing calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(4 marks each)* Determine if the series converges or diverges, justify by applying the correct test. If the series converges, find the sum.

1. $$\sum_{n=1}^{\infty} \frac{2n}{n^2+1}$$

2. $$\sum_{n=1}^{\infty} \frac{n^3+1}{n^3+n^2+1}$$

3. $$\sum_{n=1}^{\infty} \left[\frac{3}{2^n} - 4\left(\frac{1}{3}\right)^{n+1}\right]$$

(2) Lets use the n^{th} term divergence test first.

$$\lim_{n\to\infty} \frac{n^3+1}{n^3+n^2+1} = 1$$

$\therefore$ the series $\sum_{n=1}^{\infty} \frac{n^3+1}{n^3+n^2+1}$ diverge since the limit $\neq 0$ by the n^{th} term divergence test.

(1) The limit comparison test or integral test can be used. Lets try the integral test. Lets verify the conditions in order to use the integral test. Let $f(x) = \frac{2x}{x^2+1}$

Is $f(x)$ positive for $x \geq 1$? yes

Is $f(x)$ continuous for $x \geq 1$? yes

Is $f(x)$ decreasing for $x > 1$?

$$f'(x) = \frac{2(x^2+1) - 2x(2x)}{(x^2+1)^2}$$

$$= \frac{1-2x^2}{(x^2+1)}$$ ✓ negative for $x > 1$.

$$\int_1^\infty \frac{2x}{x^2+1}\,dx = \lim_{b\to\infty}\int_1^b \frac{2x}{x^2+1}\,dx$$

$$= \lim_{b\to\infty}\left[\ln|x^2+1|\right]_1^b$$

$$= \lim_{b\to\infty} \ln|b^2+1| - \ln|2| \quad \nearrow \infty$$

$\therefore$ the integral diverges $\therefore$ by the integral test the series diverge.

③ $$\sum_{n=1}^{\infty}\frac{3}{2^n} = \sum_{n=1}^{\infty}3\left(\frac{1}{2}\right)^n = \sum_{n=0}^{\infty}3\left(\frac{1}{2}\right)^n - a_0$$

$$= \frac{3}{1-\frac{1}{2}} - 3 = 6-3 = 3$$

$$\sum_{n=1}^{\infty}4\left(\frac{1}{3}\right)^{n+1} = \sum_{n=1}^{\infty}\frac{4}{3}\left(\frac{1}{3}\right)^n = \sum_{n=0}^{\infty}\frac{4}{3}\left(\frac{1}{3}\right) - a_0$$

$$= \frac{4/3}{1-\frac{1}{3}} - \frac{4}{3} = 2-\frac{4}{3} = \frac{2}{3}$$

$$\therefore \sum \frac{3}{2^n} - 4\left(\frac{1}{3}\right)^{n+1} = 3 - \frac{2}{3} = \frac{7}{3}$$