**Dawson College**: **Calculus II**: 201-NYB-05-S1: Summer 2008

**Name:** ____________________

**Student ID:** ____________________

# Quiz 4

This quiz is graded out of 12 marks. No books, graphing calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

**Question 1.** *(4 marks each)* Determine if the series converges or diverges, justify by applying the correct test. If the series converges, find the sum.

1. $$\sum_{n=1}^{\infty} \frac{1-2n}{5n+1}$$

2. $$\sum_{n=1}^{\infty} \frac{n^2+1}{\sqrt{n^5+n^2+1}}$$

3. $$\sum_{n=1}^{\infty} \left[\frac{2}{3^{n-1}} - 3\left(\frac{1}{2}\right)^n\right]$$

(1) Lets try the $n^{th}$ term divergence test

$$\lim_{n\to\infty} \frac{1-2n}{5n+1} = \frac{-2}{5}$$

$\therefore$ the series diverge since the limit is not equal to zero. by the $n^{th}$ term divergence test.

(2) Lets use $\sum_{n=1}^{\infty} \frac{n^2}{\sqrt{n^5}}$ as a test series

$\sum_{n=1}^{\infty} \frac{1}{n^{1/2}} = \sum_{n=1}^{\infty} \frac{n^2}{\sqrt{n^5}}$, the test series diverges since where $\frac{1}{2} = p \leq 1$. it is a p-series

$$\lim_{n\to\infty} \frac{n^2}{\sqrt{n^5}} \frac{\sqrt{n^5+n^2+1}}{n^2+1} = \lim_{n\to\infty} \frac{n^2}{n^2+1} \frac{\sqrt{n^5+n^2+1}}{\sqrt{n^5}}$$

$$= \lim_{n\to\infty} \frac{n^2}{n^2+1} \sqrt{\frac{n^5+n^2+1}{n^5}}$$

$$= \lim_{n\to\infty} \frac{\cancel{n^2}/\cancel{n^2}}{\cancel{n^2}/\cancel{n^2} + \cancel{1/n^2}^{\,0}} \sqrt{1 + \cancel{\frac{n^2}{n^5}}^{\,0} + \cancel{\frac{1}{n^5}}^{\,0}}$$

$$= 1$$

$\therefore$ the limit is positive and finite, the series diverge.

③ $\sum_{n=1}^{\infty} \frac{2}{3^{n-1}} = \sum_{n=1}^{\infty} 6\left(\frac{1}{3}\right)^n$ $\therefore$ converges

$$= \sum_{n=0}^{\infty} 6\left(\frac{1}{3}\right)^n - a_0$$

$$= \frac{6}{1-1/3} - 6$$

$$= \frac{6}{2/3} - 6 = 3$$

$\sum_{n=1}^{\infty} 3\left(\frac{1}{2}\right)^n = \sum_{n=0}^{n} 3\left(\frac{1}{2}\right)^n - a_0$ $\therefore$ converges

$$= \frac{3}{1-1/2} - 3 = 3$$

$$\therefore \sum_{n=1}^{\infty} \frac{2}{3^{n-1}} - 3\left(\frac{1}{2}\right)^n = 3 - 3 = 0$$