Dawson College: **Calculus II**: 201-NYB-05-S1: Summer 2008

Name: ____________________
Student ID: ____________________

Test 1

This test is graded out of 45 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Formula:

$\sum_{i=1}^{n} c = cn$ where c is a constant $\qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

Question 1. *(3 marks)* Integrate the following indefinite integral:

$$\int \frac{1}{\sqrt[9]{x}} + \sqrt[9]{x} + \csc x \, dx = \frac{9x^{8/9}}{8} + \frac{9x^{10/9}}{10} - \ln|\csc x + \cot x| + c$$

Question 2. *(5 marks)* Evaluate the definite integral using first principles *(i.e. limit process)*:

$$\int_0^2 x^2 + 2x \, dx \qquad \Delta x = \frac{b-a}{n} = \frac{2}{n}, \quad x_i = a + i\Delta x = \frac{2i}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\left(\frac{2i}{n}\right)^2 + 2\left(\frac{2i}{n}\right)\right)\frac{2}{n}$$

$$= \lim_{n\to\infty} \frac{2}{n}\left[\sum_{i=1}^{n}\left(\frac{4i^2}{n^2}\right) + \sum_{i=1}^{n}\frac{4i}{n}\right]$$

$$= \lim_{n\to\infty} \frac{8}{n^3}\sum_{i=1}^{n} i^2 + \frac{4}{n^2}\sum_{i=1}^{n} i$$

$$= \lim_{n\to\infty} \frac{8}{n^{\cancel{3}2}}\frac{\cancel{n}(n+1)(2n+1)}{6} + \frac{8}{n^{\cancel{2}}}\frac{\cancel{n}(n+1)}{2}$$

$$= \lim_{n\to\infty} \frac{16\cancel{n^2}}{6\cancel{n^2}} + \overset{0}{\cancel{\frac{24n}{6n^2}}} + \overset{0}{\cancel{\frac{8}{6n^2}}} + \frac{48n}{2n} + \overset{0}{\cancel{\frac{4}{2n}}}$$

$$= \frac{16}{6} + 4 = \frac{20}{3}$$

Question 3. *(5 marks)* Integrate the following indefinite integral:

$$\int \frac{1}{\sqrt{x}\sqrt{1-x}}\,dx = \int \frac{1}{\sqrt{x}\sqrt{1-(\sqrt{x})^2}}\,dx$$

① $u = \sqrt{x}$

$du = \dfrac{dx}{2\sqrt{x}}$

② $2\sqrt{x}\,du = dx$

$$\overset{①,②}{=} \int \frac{1}{\cancel{\sqrt{x}}(\sqrt{1-(\ )^2}}\,\cancel{2\sqrt{x}}du$$

$$= 2\int \frac{1}{\sqrt{1-u^2}}\,du$$

$$= 2\arcsin u + C$$

$$\overset{①}{=} 2\arcsin\sqrt{x} + C$$

Question 4. *(5 marks)* Integrate the following indefinite integral:

$$\int \tan(\cot 2x)\csc^2 2x\,dx = \int \tan u\,\cancel{\csc^2 2x}\,\frac{du}{-2\cancel{\csc^2 2x}}$$

$u = \cot 2x$

$du = -\csc^2 2x\,(2)\,dx$

$\dfrac{du}{-2\csc^2 2x} = dx$

$$= -\frac{1}{2}\int \tan u\,du$$

$$= \frac{1}{2}\ln|\cos u| + C$$

$$\overset{①}{=} \frac{1}{2}\ln|\cos\cot 2x| + C$$

Question 5. Given $\int_a^b f(x)\,dx = 3$, $\int_a^c g(x)\,dx = 3$ and $\int_b^c f(x)\,dx = 4$ evaluate the following definite integrals:

1. *(1 mark)*

$$\int_a^a 6f(x)\,dx = 6\int_a^a f(x)\,dx = 0$$

2. *(3 marks)*

$$\int_c^a f(x) - 2g(x)\,dx = \int_c^a f(x)\,dx - 2\int_c^a g(x)\,dx$$

$$= -\left[\int_a^b f(x)\,dx + \int_b^c f(x)\right] + 2\int_a^c g(x)\,dx$$

$$= -\left[3+4\right] + 2(3) = -1$$

Question 6. *(5 marks)* Evaluate the following definite integral:

$$\int_0^{\pi/4} \tan x \sec^2 x\,dx \overset{①,②}{=} \int_0^1 u\,\cancel{\sec^2 x}\,\frac{du}{\cancel{\sec^2 x}}$$

① $u = \tan x$

$du = \sec^2 x\,dx$

② $\dfrac{du}{\sec^2 x} = dx$

$u(0) = \tan(0)$

$= 0$

$u(\pi/4) = \tan \pi/4$

$= 1$

$$= \int_0^1 u\,du$$

$$= \left[\frac{u^2}{2}\right]_0^1$$

$$= \frac{1}{2}$$

Question 7. *(3 marks)* Use the Second Fundamental Theorem of Calculus to find $F'(x)$.

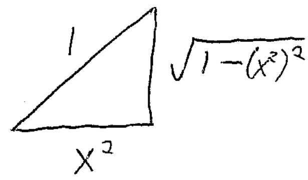
$f(g(x)) = F(x) = \int_0^{\cos x^2} \arctan y \, dy$

Let $g(x) = \cos x^2 \Rightarrow g'(x) = -\sin x^2 (2x) = -2x \sin x^2$

$f(x) = \int_0^x \arctan y \, dy \Rightarrow f'(x) = \arctan x$

(by the second fundamental theorem of calculus)

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))\,g'(x)$$
$$= \arctan \cos x^2 \left(-2x \sin x^2\right)$$
$$= -2x \sin x^2 \arctan \cos x^2$$
$$= -2\cancel{x} \sin x^2 \frac{\sqrt{1-x^4}}{x^{\cancel{2}}}$$
$$= \frac{-2 \sin x^2 \sqrt{1-x^4}}{x}$$



Question 8. *(5 marks)* Integrate the following indefinite integral:

$$\int \frac{x^2 - 4x}{x^2}\, dx = \int \frac{\cancel{x^2}}{\cancel{x^2}} - \frac{4\cancel{x}}{x^{\cancel{2}}}\, dy$$
$$= \int 1 - \frac{4}{x}\, dx$$
$$= x - 4\ln|x| + C$$

Question 9. *(5 marks)* Integrate the following indefinite integral:

$$\int e^{\cos 3x} \sin 3x \, dx$$

$\overset{①,②}{=} \int e^{u} \cancel{\sin 3x} \frac{du}{-3\cancel{\sin 3x}}$

① $u = \cos 3x$

$du = -\sin 3x \, (3) \, dx$

② $\frac{du}{-3\sin 3x} = dx$

$= -\frac{1}{3} \int e^{u} \, du$

$= -\frac{1}{3} e^{u} + C$

$\overset{①}{=} \frac{-e^{\cos 3x}}{3} + C$

Question 10. *(5 marks)* Evaluate the following definite integral:

$$\int_1^2 (x^2+x)(2x^3+3x^2)^2 \, dx$$

$\overset{①,②}{=} \int_5^{28} \cancel{(x^2+x)} u^2 \frac{du}{6\cancel{(x^2+x)}}$

① $u = 2x^3 + 3x^2$

$du = 6x^2 + 6x \, dx$

② $dx = \frac{du}{6(x^2+x)}$

$= \frac{1}{6} \int_5^{28} u^2 \, du$

$u(1) = 2(1)^3 + 3(1)^2$

$= 5$

$u(2) = 2(2)^3 + 3(2)^2$

$= 28$

$= \frac{1}{6} \left[\frac{u^3}{3} \right]_5^{28}$

$= \frac{28^3}{18} - \frac{5^3}{18}$

$= \frac{21827}{18}$

Bonus Question. *(3 marks)*

Integrate the following indefinite intergral:

$$\int \frac{1}{(\arctan x)(\ln \arctan x)(1+x^2)} \, dx$$

$\overset{①,②}{=} \int \frac{1}{u} \, du$

① $u = \ln \arctan x$

② $du = \frac{1}{\arctan (1+x^2)} dx$

$= \ln|u| + C$

$\overset{①}{=} \ln|\ln \arctan (1+x^2)| + C$