LOGARITHMS – EXERCISE – ANSWERS

(1) 1 (2) 0 (3) 5 (4) 4 (5) 1/22 (6) 1 (7) 0 (8) 9 (9) 81 (10) 1/9

(11) -3 (12) 392 (13) (a) 7 (b) 3 (c) 9 (14) (a) 3 (b) 1 (c) 2 (15) $\log_5 a + \log_5 b - \log_5 c$

(16) $\frac{1}{2}\log_n x - \frac{1}{2}$ (17) $4\log_7 x - 2$ (18) $\log 2 + \log \pi + \frac{\log a}{2} - \frac{\log b}{2}$

(19) $2\log_5 a + 3\log_5 b - 2$ (20) $2\log_b a + 3 + \frac{1}{2}\log_b c$ (21) $1 + \log_a(a+1)$

(22) $\log a - 1 - \frac{3}{2}\log b$ (23) $5\log a - \frac{1}{2}\log b - 3\log c$ (24) $2 + \ln(x+y) + \ln(x-y)$

(25) $\frac{2}{n} + \frac{3}{n}\log x - \frac{m}{n}\log y$ (26) $\frac{\ln x}{3} + \frac{\ln y}{6}$ (27) $\log x^2\sqrt{y}$ (28) $\log_a\left(\frac{M^5}{N^7}\right)$

(29) $\sqrt{\frac{x}{y^3}}$ (30) $\log\frac{\sqrt{6}}{25}$ (31) $\log\frac{1}{8000}$ (32) $\log\sqrt{\frac{x^2}{y^5}}$ (33) $\ln 14$ (34) $\ln x$

(35) $\log\left(\frac{x}{10}\right)$ (36) $\log\sqrt{\frac{8x}{y^3}}$ (37) $\ln(x+1)$ (38) $\log_3(x+3)$ (39) 8 (40) 1

(41) 0.65 (42) (a) $3y$ (b) $3x+2y$ (c) $\frac{x-y}{2}$ (43) (a) 0.7781 (b) 0.9030 (c) 0.1509

(43) (d) 1.0791 (e) 0.3522 (f) 0.6990 (g) 0.7386 (44) (a) $4y + \frac{x}{2}$ (b) $\frac{3x}{2} - \frac{5y}{2}$

(45) 1.7304 (46) 2.0166 (47) 2.3980 (48) 0.1235 (49) 1.0653 (50) 26.4400 (51) 2.5915

(52) 1.5850 (53) (a) 1.2091 (b) 124 (54) 1 (55) 5 (56) 0 (57) 3 (58) 9 (59) 1/3

(60) 3 (61) 3 (62) 2 (63) -1/2 (64) 1 (65) 1 (66) 3 (67) 20 (68) 3 (69) 3 (70) 1/4

(71) 5 (72) 5 (73) 0 (74) 10 (75) 22 (76) 11 (77) 2 (78) 7 (79) -1 (80) 5 (81) 3, 4

(82) 9 (83) 10 (84) 2, 5 (85) 2 (86) $\frac{3+\sqrt{17}}{2}$ (87) $\frac{e+1}{e-1}$ (88) 0, 2 (89) $\frac{1\pm\sqrt{5}}{2}$

(90) 6 (91) 5.8574 (92) 2.7268 (93) 2.3717 (94) 0.4343 (95) 3.2875

(96) -1.3403 (97) 2.5237 (98) 24.7808 (99) $\pm$1.0878 (100) 6.0910 (101) 3.1507

(102) 4.4185 (103) 6.3848 (104) -1.8251 (105) -0.2144 (106) -0.6842

(107) 0, 1.0586 (108) 26.1757 (109) 1.4992 (110) 20.85 (111) 6.67 (112) 0.9581

(113) 0.8401 (114) -0.5493 (115) $\frac{\ln(1/2)}{-k}$ (116) 1, 1/64 (117) 3, 15 (118) 82, 4/3

(119) 0, 1 (120) 0 (121) 1, 0.6309 (122) $\pm$1.1987 (123) 1/2, 0.7925

(124) (a) 2 million (b) 2.54 million (c) in 2020

(125) (a) 0.0162 (b) in 2013

(126) -0.00012

(127) 13.6 yrs.

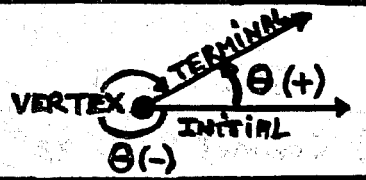
(128) (a) 0.198 (b) 55.7

(129) 14.4 grams

ANGLES IN TRIGONOMETRY

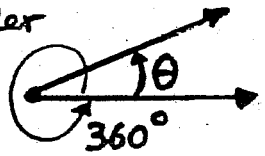
THE DEFINITION OF AN ANGLE

An **ANGLE**, θ, is the amount of a revolution that a line makes in moving from its **INITIAL** side to its **TERMINAL** side.



ANGLES IN DEGREES

Consider



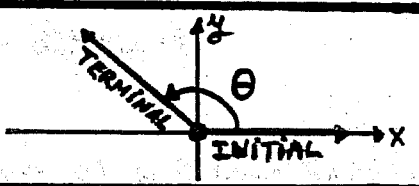
An **ANGLE**, θ, in **DEGREES**, (°), is measured as a proportion of 1 complete revolution (360°)

Also, 1° = 60 minutes (60')

TYPES OF ANGLES

ANGLES IN STANDARD POSITION

An ANGLE, θ, in **STANDARD POSITION** has its VERTEX at the origin of the rectangular coordinate system, as shown here:



NAMES OF ANGLES

RIGHT ANGLE: $90°$ **STRAIGHT ANGLE**: $180°$

COMPLEMENTARY ANGLES: TWO ANGLES WHOSE SUM IS $90°$

SUPPLEMENTARY ANGLES: TWO ANGLES WHOSE SUM IS $180°$

ACUTE ANGLE, θ: $0° < \theta < 90°$

OBTUSE ANGLE, θ: $90° < \theta < 180°$

REFLEX ANGLE, θ: $180° < \theta < 360°$

QUADRANTAL ANGLES: $0°, 90°, 180°, 270°, 360°$

CO-TERMINAL ANGLES

ANGLES θ_1 and θ_2 are **CO-TERMINAL ANGLES** if they have the same initial and terminal side. That is, if:

$$\theta_1 = \theta_2 + k \cdot 360° \quad , k \in \mathbb{I}$$

ANGLES IN TRIGONOMETRY – EXAMPLES

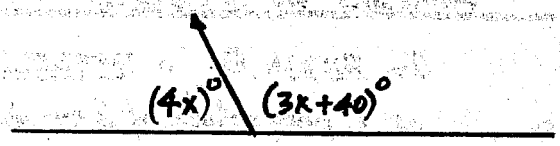
(1) Convert $75°39'$ to decimal degree measure.

$$75°39' = 75° + \frac{39}{60}° = 75° + 0.65° \Rightarrow 75.65°$$

(2) Convert $150.25°$ to degrees and minutes.

$$150.25° = 150° + 0.25° = 150° + 0.25(60') = 150° + 15' \Rightarrow 150°15'$$

(3) Find x :



consider $4x + (3x + 40) = 180$, supplementary angles

$$7x + 40 = 160$$
$$7x = 140$$
$$\therefore x = 20$$

hence the angles are: $4(20) = 80°$, and

$$3(20) + 40 = 100°$$

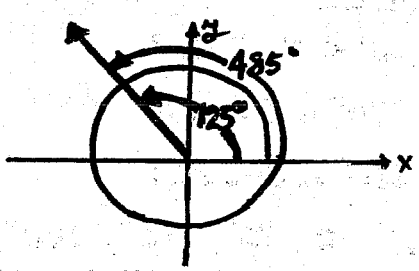
(4) what angle θ $(0° \leq \theta < 360°)$ is co-terminal to $485°$?

consider that $485° - 360° = 125°$

$$\therefore 485° = 125° + 1(360°)$$

hence $125°$ is co-terminal to $485°$

Note:



ANGLES IN TRIGONOMETRY – EXERCISES

(1) What is the degree measure of an angle that is the following proportion of a complete revolution?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) $\frac{1}{8}$ (e) $\frac{1}{12}$ (f) $\frac{1}{6}$

(2) What $(-)^\circ$ angle is equivalent to: (a) 350° (b) 260° (c) 170° (d) 50° ?

(3) What $(+)^\circ$ angle is equivalent to: (a) -25° (b) -150° (c) -200° (d) -300° ?

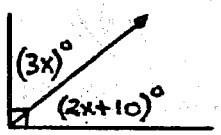
(4) Convert to decimal degree measure:

(a) $50^\circ 30'$ (b) $100^\circ 15'$ (c) $200^\circ 42'$ (d) $300^\circ 54'$

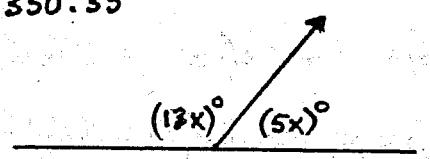
(5) Convert to degrees and minutes measure:

(a) 75.50° (b) 125.45° (c) 250.75° (d) 350.35°

(6) Find x: (a)



(b)



(7) What angle is complementary to: (a) 30° (b) 80° (c) 55°

(8) What angle is supplementary to: (a) 100° (b) 30° (c) 155°

(9) State the name of the angle: (a) 45° (b) 150° (c) 300° (d) 30° and 60° (e) 100° and 80° (f) 300° and 660° (g) 50° and -670°

(10) What angle θ $(0^\circ \le \theta < 360^\circ)$ is co-terminal to:

(a) 390° (b) 1140° (c) 535° (d) 1215° (e) 560° (f) 1020° (g) 450° (h) 1350°

(11) Find the smallest angle between the hands of a clock when the time is:

(a) 3:30 (b) 1:45

ANSWERS

(1) (a) 90° (b) 180° (c) 270° (d) 45° (e) 30° (f) 60°

(2) (a) -10° (b) -100° (c) -190° (d) -310°

(3) (a) 335° (b) 210° (c) 160° (d) 60°

(4) (a) 50.50° (b) 100.25° (c) 200.70° (d) 300.90°

(5) (a) $75^\circ 30'$ (b) $125^\circ 27'$ (c) $250^\circ 45'$ (d) $350^\circ 21'$

(6) (a) $x = 16$ (b) $x = 10$ (7) (a) 60° (b) 10° (c) 35° (8) (a) 80° (b) 150° (c) 25°

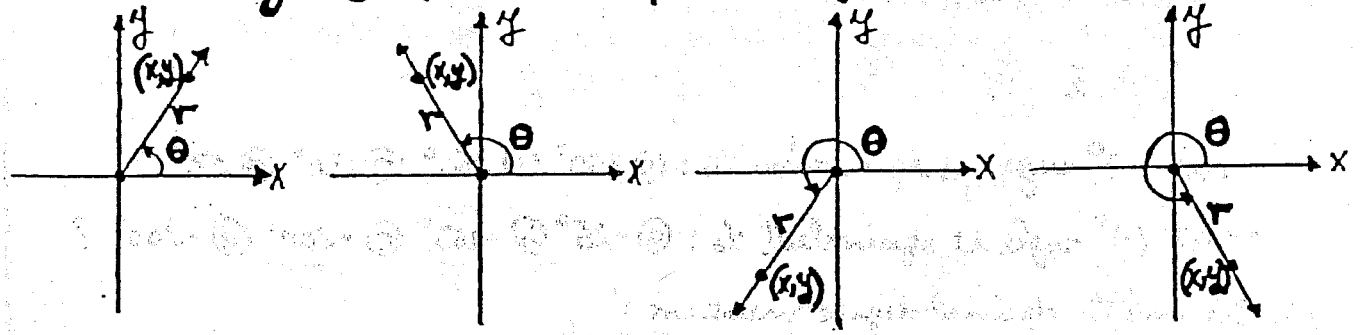
(9) (a) acute (b) obtuse (c) reflex (d) complementary (e) supplementary (f) co-terminal (g) co-terminal

(10) (a) 30° (b) 60° (c) 175° (d) 135° (e) 200° (f) 300° (g) 90° (h) 270°

(11) (a) 75° (b) 142.50°

THE TRIGONOMETRIC FUNCTIONS

Consider an angle θ in standard position by quadrant, as follows:

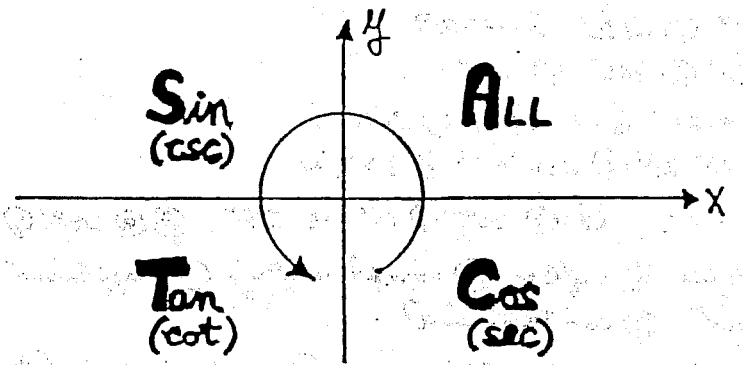


where $r=\sqrt{(x-0)^2+(y-0)^2}=\sqrt{x^2+y^2}$; $x,y,r\in\mathbb{R}, r>0$ and $x,y\neq 0$ as required to define:

DEFINITIONS OF THE TRIGONOMETRIC FUNCTIONS	DOMAIN	RANGE
$f(\theta)=$ sine of $\theta = \sin\theta=\frac{y}{r}$	all θ	$f(\theta)\in[-1,1]$
$f(\theta)=$ cosecant of $\theta = \csc\theta=\frac{r}{y}$ *	all $\theta, \neq 0°, \pm 180°, \cdots$	$f(\theta)\notin(-1,1)$
$f(\theta)=$ cosine of $\theta = \cos\theta=\frac{x}{r}$	all θ	$f(\theta)\in[-1,1]$
$f(\theta)=$ secant of $\theta = \sec\theta=\frac{r}{x}$ *	all $\theta, \neq \pm 90°, \pm 270°, \cdots$	$f(\theta)\notin(-1,1)$
$f(\theta)=$ tangent of $\theta = \tan\theta=\frac{y}{x}$ *	all $\theta, \neq \pm 90°, \pm 270°, \cdots$	$f(\theta)\in\mathbb{R}$
$f(\theta)=$ cotangent of $\theta = \cot\theta=\frac{x}{y}$	all $\theta, \neq 0°, \pm 180°, \cdots$	$f(\theta)\in\mathbb{R}$

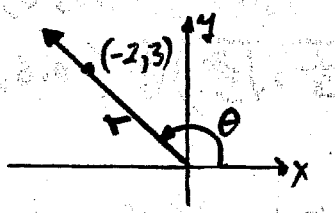
* reciprocal functions

THE CAST RULE FOR POSITIVE VALUES



THE TRIGONOMETRIC FUNCTIONS – EXAMPLES

(1) Consider an angle θ in standard position with the point $(-2,3)$ on its terminal side. Find the values of the 6 trigonometric functions of θ.

consider 

then $x=-2$, $y=3$, and

$r=\sqrt{x^2+y^2}$

$r=\sqrt{(-2)^2+3^2}=\sqrt{4+9}$

$\therefore r=\sqrt{13}$

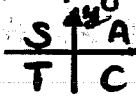
hence

$\begin{cases}\sin\theta=\frac{y}{r}=\frac{3}{\sqrt{13}}=\frac{3\sqrt{13}}{13} & \text{, rationalized denominator}\\ \csc\theta=\frac{r}{y}=\frac{\sqrt{13}}{3}\end{cases}$

$\begin{cases}\cos\theta=\frac{x}{r}=\frac{-2}{\sqrt{13}}=-\frac{2\sqrt{13}}{13}\\ \sec\theta=\frac{r}{x}=\frac{\sqrt{13}}{-2}=-\frac{\sqrt{13}}{2}\end{cases}$

$\begin{cases}\tan\theta=\frac{y}{x}=\frac{3}{-2}=-\frac{3}{2}\\ \cot\theta=\frac{x}{y}=\frac{-2}{3}=-\frac{2}{3}\end{cases}$

(2) Determine the quadrant of an angle θ if $\sin\theta>0$ and $\cos\theta<0$.

consider 

$\therefore \sin\theta>0 \Rightarrow \theta\in Q_1$ or Q_2

and, $\cos\theta<0 \Rightarrow \theta\in Q_2$ or Q_4

} hence $\theta\in Q_2$

(3) If $\cos\theta=-\frac{2}{5}$ and $\tan\theta>0$, then find $\sin\theta$.

By the CAST rule $\theta\in Q_3$, thus $\cos\theta=\frac{x}{r}=-\frac{2}{5}=\frac{-2}{5}$

$\therefore x=-2$ and $r=5$

then consider

$r=\sqrt{x^2+y^2}$

$5=\sqrt{(-2)^2+y^2}$

$5=\sqrt{4+y^2}$

$25=4+y^2$

$21=y^2$

$\therefore \pm\sqrt{21}=y$, thus $y=-\sqrt{21}$, since $\theta\in Q_3$

hence $\sin\theta=\frac{y}{r}=\frac{-\sqrt{21}}{5}$

$\therefore \sin\theta=-\frac{\sqrt{21}}{5}$

THE TRIGONOMETRIC FUNCTIONS EXERCISES

(1) Consider an angle θ in standard position with the given (x,y) point on its terminal side. Then find the values of the 6 Trigonometric Functions of θ.

(a) $(1,2)$ (b) $(5,12)$ (c) $(4,3)$ (d) $(4,5)$ (e) $(-5,12)$ (f) $(15,-8)$ (g) $(-2,-3)$

(h) $(8,-15)$ (i) $(3,-4)$ (j) $(-4,-5)$ (k) $\left(\sqrt{\frac{1}{3}}, \sqrt{\frac{2}{3}}\right)$ (l) $(-0.6, 0.8)$

(m) $\left(-\frac{1}{3}, \frac{2\sqrt{2}}{3}\right)$

(2) Consider an angle θ in standard position with the given (x,y) point on its terminal side. Then find the required.

(a) $(-3,4)$; $\tan\theta$ (b) $(-3,4)$; $\sin\theta+\cos\theta$ (c) $(-3,-4)$; $\sin\theta-\cos\theta$

(d) $(-2,3)$; $\sin\theta+\csc\theta$ (e) $(2,-5)$; $6\cos\theta+5\sin\theta$ (f) $(-3,7)$; $6\sin\theta\cdot\cos\theta$

(3) Consider an angle θ in standard position. Then find the quadrant that its terminal side lies in, if:

(a) $\cot\theta<0$ and $\sin\theta>0$ (b) $\tan\theta<0$ and $\cos\theta<0$

(c) $\sin\theta<0$ and $\tan\theta<0$ (d) $\sec\theta<0$ and $\cot\theta<0$

(e) $\csc\theta<0$ and $\sec\theta>0$ (f) $\csc\theta>0$ and $\tan\theta<0$

(g) $\sec\theta<0$ and $\cot\theta>0$ (h) $\sin\theta>0$ and $\sec\theta<0$

(i) $\cot\theta>0$ and $\cos\theta<0$ (j) $\sin\theta>0$ and $\cos\theta>0$

(4) Find the values of the other trigonometric functions, if:

(a) $\tan\theta=\frac{3}{4}$ and $\sin\theta<0$ (b) $\cot\theta=-\frac{3}{4}$ and $\cos\theta>0$

(c) $\sec\theta=-\frac{13}{5}$ and $\tan\theta<0$ (d) $\cos\theta=-\frac{1}{2}$ and $\sin\theta>0$

(e) $\sin\theta=-\frac{2}{3}$ and $\sec\theta>0$ (f) $\sec\theta=-5$ and $\csc\theta<0$

(g) $\csc\theta=8$ and $\cos\theta<0$ (h) $\cot\theta=0$ and $\sin\theta=-1$

THE TRIGONOMETRIC FUNCTIONS EXERCISES - ANSWERS

(1)

	$\sin\theta$	$\csc\theta$	$\cos\theta$	$\sec\theta$	$\tan\theta$	$\cot\theta$
(a)	$2/\sqrt{5}$	$\sqrt{5}/2$	$1/\sqrt{5}$	$\sqrt{5}$	2	$1/2$
(b)	$12/13$	$13/12$	$5/13$	$13/5$	$12/5$	$5/12$
(c)	$3/5$	$5/3$	$4/5$	$5/4$	$3/4$	$4/3$
(d)	$5/\sqrt{41}$	$\sqrt{41}/5$	$4/\sqrt{41}$	$\sqrt{41}/4$	$5/4$	$4/5$
(e)	$12/13$	$13/12$	$-5/13$	$-13/5$	$-12/5$	$-5/12$
(f)	$-8/17$	$-17/8$	$15/17$	$17/15$	$-8/15$	$-15/8$
(g)	$-2\sqrt{13}/13$	$-\sqrt{13}/3$	$-2/\sqrt{13}$	$-\sqrt{13}/2$	$3/2$	$2/3$
(h)	$-15/17$	$-17/15$	$8/17$	$17/8$	$-15/8$	$-8/15$
(i)	$-4/5$	$-5/4$	$3/5$	$5/3$	$-4/3$	$-3/4$
(j)	$-5/\sqrt{41}$	$-\sqrt{41}/5$	$-4/\sqrt{41}$	$-\sqrt{41}/4$	$5/4$	$4/5$
(k)	$\sqrt{2}/3$	$\sqrt{3}/2$	$\sqrt{1/3}$	$\sqrt{3}$	$\sqrt{2}$	$1/\sqrt{2}$
(l)	0.8	1.25	-0.6	-1.66	-1.66	-0.75
(m)	$2\sqrt{2}/2$	$3/2\sqrt{2}$	$-1/3$	-3	$-2\sqrt{2}$	$-1/2\sqrt{2}$

(2) (a) $-\frac{4}{3}$ (b) $\frac{1}{5}$ (c) $-\frac{1}{5}$ (d) $\frac{22\sqrt{13}}{39}$ (e) $-\frac{13\sqrt{29}}{29}$ (f) $-\frac{126}{58}$

(3) (a) 2^{nd} (b) 2 (c) 4 (d) 2 (e) 4 (f) 2 (g) 3 (h) 2 (i) 3 (j) 1

(4) (a) $\sin\theta = -\frac{3}{5}$, $\cos\theta = -\frac{4}{5}$, $\csc\theta = -\frac{5}{3}$, $\sec\theta = -\frac{5}{4}$, $\cot\theta = \frac{4}{5}$

(b) $\sin\theta = -\frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $\csc\theta = -\frac{5}{4}$, $\sec\theta = \frac{5}{3}$, $\tan\theta = -\frac{4}{3}$

(c) $\sin\theta = \frac{12}{13}$, $\cos\theta = -\frac{5}{13}$, $\csc\theta = \frac{13}{12}$, $\tan\theta = -\frac{12}{5}$, $\cot\theta = -\frac{5}{12}$

(d) $\sin\theta = \sqrt{3}/2$, $\csc\theta = 2/\sqrt{3}$, $\sec\theta = -2$, $\tan\theta = -\sqrt{3}$, $\cot\theta = -1/\sqrt{3}$

(e) $\cos\theta = \sqrt{5}/3$, $\sec\theta = 3/\sqrt{5}$, $\csc\theta = -3/2$, $\tan\theta = -2/\sqrt{5}$, $\cot\theta = -\sqrt{5}/2$

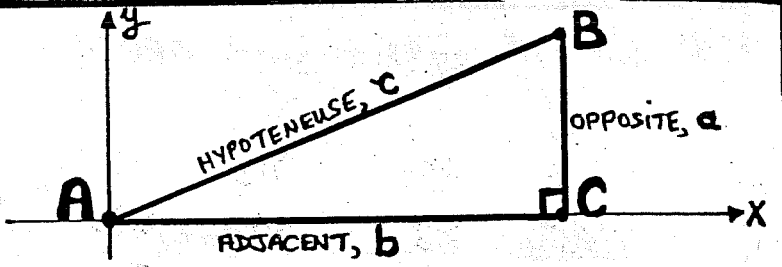
(f) $\sin\theta = -\frac{\sqrt{24}}{5}$, $\cos\theta = -\frac{1}{5}$, $\csc\theta = -\frac{5}{\sqrt{24}}$, $\tan\theta = \sqrt{24}$, $\cot\theta = 1/\sqrt{24}$

(g) $\sin\theta = 1/8$, $\cos\theta = -\frac{\sqrt{63}}{8}$, $\sec\theta = -\frac{8}{\sqrt{63}}$, $\tan\theta = -\frac{1}{\sqrt{63}}$, $\cot\theta = -\sqrt{63}$

(h) $\cos\theta = 0$, $\sec\theta$ undefined, $\csc\theta = -1$, $\tan\theta$ undefined

SOLVING RIGHT TRIANGLES

Consider the right triangle ABC with the acute angle A in standard position.



$$c^2 = a^2 + b^2$$

$$A + B = 90^\circ$$

$$\left\{ \sin A = \frac{a}{c} = \frac{\text{OPPOSITE}}{\text{HYPOTENEUSE}} \right\}$$

$$\left\{ \csc A = \frac{c}{a} = \frac{\text{HYPOTENEUSE}}{\text{OPPOSITE}} \right\}$$

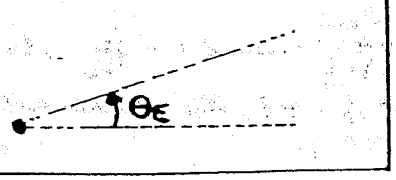
$$\left\{ \cos A = \frac{b}{c} = \frac{\text{ADJACENT}}{\text{HYPOTENEUSE}} \right\}$$

$$\left\{ \sec A = \frac{c}{b} = \frac{\text{HYPOTENEUSE}}{\text{ADJACENT}} \right\}$$

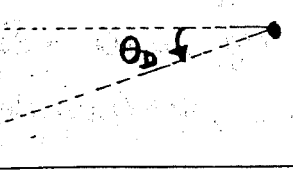
$$\left\{ \tan A = \frac{a}{b} = \frac{\text{OPPOSITE}}{\text{ADJACENT}} \right\}$$

$$\left\{ \cot A = \frac{b}{a} = \frac{\text{ADJACENT}}{\text{OPPOSITE}} \right\}$$

ANGLES OF ELEVATION, θ_E



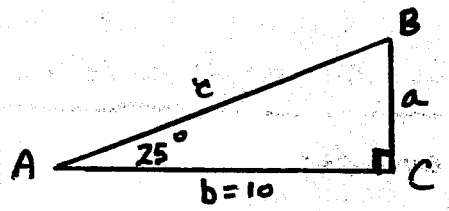
ANGLES OF DEPRESSION, θ_D



NOTE: We may use the mnemonic device **SOHCAHTOA** for the Sin, Cos, and Tan ratios.

SOLVING RIGHT TRIANGLES - EXAMPLES

(1) Solve the right triangle ABC:



Solution: consider $A+B=90^\circ$

$25^\circ+B=90^\circ$

$\therefore B=65^\circ$

consider $\tan A=\frac{a}{b}$

$\tan 25^\circ=\frac{a}{10}$

$10\tan 25^\circ=a$

$\therefore 4.66\simeq a$

and consider $\cos A=\frac{b}{c}$

$\cos 25^\circ=\frac{10}{c}$

$c\cos 25^\circ=10$

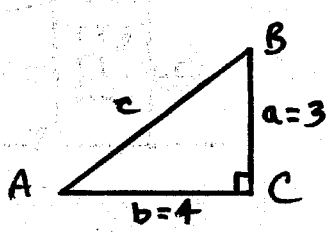
$c=\frac{10}{\cos 25^\circ}$

$\therefore c\simeq 11.03$

Note: (1) To check the solution consider the Pythagorean Theorem.

(2) For a rough check, consider side-angle proportionality (opposite the smallest angle is the smallest side, etc)

(2) Solve the right triangle ABC:



Solution: consider $c^2=a^2+b^2$

$c^2=3^2+4^2$

$c^2=9+16$

$c^2=25$

$c=\sqrt{25}$

$\therefore c=5$

consider $\tan A=\frac{a}{b}$

$\tan A=\frac{3}{4}$

$\therefore \tan A=0.75$

hence $A\simeq 36.87^\circ$, $\tan^{-1}$ in calculator

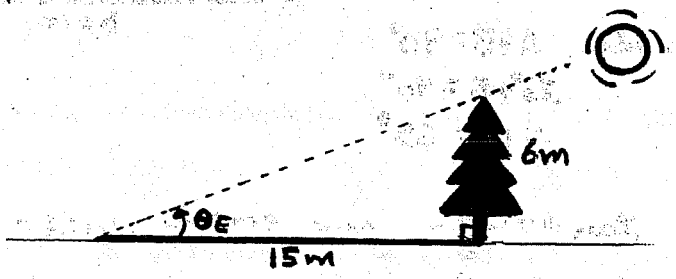
consider $A+B=90^\circ$

$36.87^\circ+B=90^\circ$

$\therefore B=53.13^\circ$

(3) Find the angle of elevation of the sun when a 6 meter tall tree casts a 15 meter long shadow.

consider

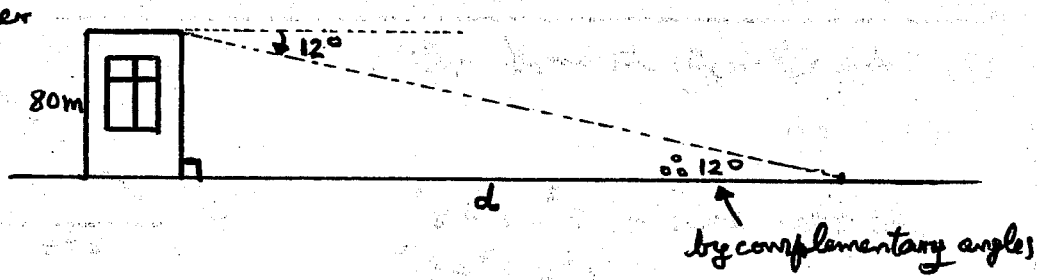


then $\tan\theta_E = \frac{6}{15}$

$\therefore \theta_E \approx 21.80°$

(4) From the top of an 80 meter high building the angle of depression to a point on the ground is 12°. How far is the point from the base of the building?

consider



by complementary angles

then $\tan 12° = \frac{80}{d}$

$d \tan 12° = 80$

$d = \frac{80}{\tan 12°}$

$\therefore d \approx 376.4$ meters

SOLVING RIGHT TRIANGLES — EXERCISES

(1) For each triangle ABC ($C = 90°$) find $\sin A$, $\cos A$, $\tan A$, $\sin B$, $\cos B$ and $\tan B$, given:

(a) $a = 3, b = 4$ (b) $a = 7, c = 25$ (c) $b = 8, c = 17$

(d) $a = 8, b = 11$ (e) $b = 49, c = 86$ (f) $a = 59, c = 78$

(2) Use a calculator to find a decimal approximation to each value.

(a) $\sin 82°$ (b) $\cos 31°$ (c) $\tan 59°$ (d) $\sin 27°$ (e) $\cos 66°$ (f) $\tan 43°$

(3) Use a calculator to find a decimal approximation for the acute angle A.

(a) $\sin A = 0.80$ (b) $\cos A = 0.20$ (c) $\tan A = 0.75$

(d) $\tan A = 1.4740$ (e) $\sin A = 0.4317$ (f) $\cos A = 0.5978$

(4) Solve the right triangle ABC ($C = 90°$) given:

(a) $b = 4, A = 73°$ (b) $a = 6, A = 29°$ (c) $A = 70°, a = 100$ (d) $A = 55°, c = 20$

(e) $c = 40, B = 20°$ (f) $B = 49°, a = 3$ (g) $c = 14.92, B = 52°$ (h) $B = 58°, b = 7$

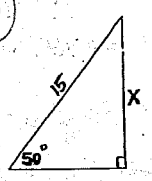
(5) Solve the right triangle ABC ($C = 90°$) given:

(a) $a = 7, b = 8$ (b) $b = 12, c = 15$ (c) $a = 14, b = 39$ (d) $a = 24, c = 25$

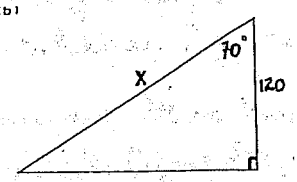
(e) $a = 27, b = 8$ (f) $a = 38, b = 63$ (g) $b = 26, c = 54$ (h) $a = 569, b = 341$

(6) Find x in each case.

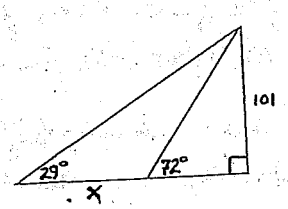
(a)



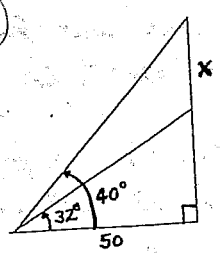
(b)



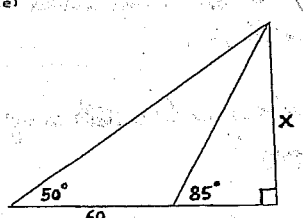
(c)



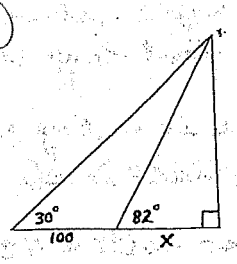
(d)



(e)



(f)



SOLVING RIGHT TRIANGLES – EXERCISES

(7) A ladder leaning against a wall makes an angle of 63° with the ground. If the ladder is 6.7 meters long, how high up the wall does it reach?

(8) a 3.6 meter long ladder makes an angle of 22.50° with a wall. How far is the foot of the ladder from the wall?

(9) a 6 m long ladder is placed against a wall so that its foot is 1.5 m from the wall. What angle does the ladder make with the wall?

(10) For maximum safety the distance between the foot of a ladder and a wall should be $\frac{1}{4}$ of the length of the ladder. What angle does a ladder make with the ground when it is in its safest position?

(11) An escalator from the first floor of a building to the second floor is 20 feet long and makes an angle of 35° with the floors. What is the vertical distance between the floors?

(12) A 6.7 m long escalator runs from the first to the second floor of a building. If the vertical distance between the floors is 3.2 m, find the angle that the escalator makes with the floors.

(13) a support cable from the ground to the top of a 29 m tall telephone pole makes an angle of 41° with the ground. How long is the cable?

(14) A guy wire 86 feet long is attached to the top of an antenna mast that is 73 feet high. What is the angle that the wire makes with the mast?

(15) From a point on the ground that is 1750 feet from the base of a building, the angle of elevation to the top of the building is 32°. What is the height of the building?

(16) An observer measures the angle of elevation from the ground to the top of a 25 m tall tree to be 75°. How far is he from the base of the tree?

(17) When a boat is 67.5 m from shore the angle of elevation to the top of a lighthouse is 15°. What is the height of the lighthouse?

(18) What is the angle of elevation of the sun when a 6 foot tall person casts a 10 foot long shadow?

(19) A tree casts a 20 m long shadow when the angle of elevation of the sun is 29°. How tall is the tree?

(20) The CN tower casts a 1122 m long shadow when the angle of elevation of the sun is 26°14'. What is the height of the tower?

SOLVING RIGHT TRIANGLES - EXERCISES

(21) A ground level observer, 5.2 km from the launch pad, observes a missile's vertical ascent.

(a) How high is the missile when the observer's angle of elevation to it is 31°27'?

(b) What will the angle of elevation be when the missile reaches a height of 30 km?

(22) A man measures the shadow of a building to be 48 m long. At that time his own shadow is 210 cm long. If the man is 175 cm tall, what is the height of the building?

(23) From the top of a 50 foot tall observation tower, the angle of depression to an object on the ground is 15.7°. How far along the ground is the object from the base of the tower?

(24) From the top of a building the angle of depression to a parked car is 29.17°. If the building is 25.5 m high, how far is the car from the base of the building?

(25) A surveyor stands on a 50 foot high cliff directly above one bank of a river. From there, the angle of depression to the opposite bank is 10°. How wide is the river?

(26) A pilot in an airplane at an altitude of 4000 feet observes the angle of depression to an airport to be 12°. How far is the airport from a point on the ground directly below the plane?

(27) An airplane approaching an airport plans to begin its descent, with an angle of depression of 3°, when its ground distance from the airport is 50 km. What should its altitude be then?

(28) From the top of a 12 m high building the angle of elevation to the top of a taller building is 17°. If the buildings are 25 m apart, what is the height of the taller building?

SOLVING RIGHT TRIANGLES – EXERCISES

(29) From the top of a building, the angle of elevation to the top of a taller building is 30°, and the angle of depression to its base is 15°. If the buildings are 100 m apart, find the height of each building.

(30) How tall is a bridge support column if at water level 10 ft. from the column it is determined that the angle of elevation to the top of the column is 80° and the angle of depression to its underwater base is 50°?

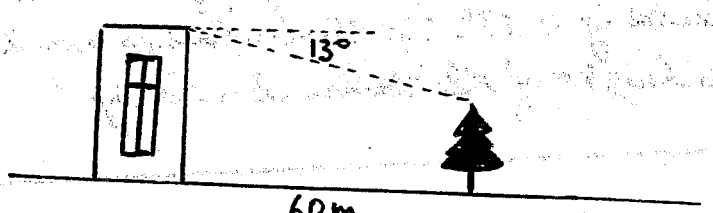
(31) Measured from a building window the angle of depression to the bottom of a building across the street is 37° and the angle of elevation to its top is 51°. If the street is 24 m wide, find the height of the building across the street.

(32) From a point on level ground the angle of elevation to the top of a tower is 36.2°. From a point 23.4 m closer to the tower the angle of elevation to its top is 48.8°. Find the height of the tower.

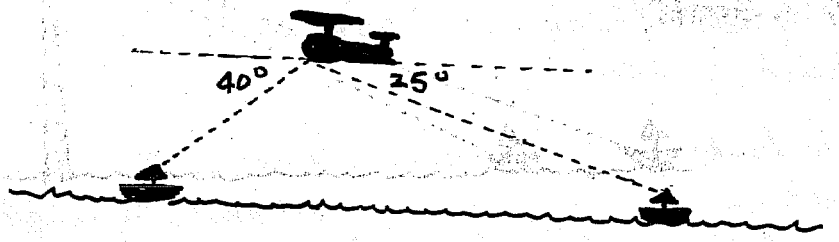
(33) A flagpole sits on the top of a building. From a point on the ground 37.5 m from the base of the building the angles of elevation to the top and bottom of the flagpole are 70° and 67° respectively. What is the height of the flagpole?

(34) A 7.5 m tall lighthouse stands on the top of seashore cliff. From a point offshore the angles of elevation to the top and bottom of the lighthouse are 47.20° and 45.22° respectively. Find the height of the cliff.

(35) Find the height of the tree in the picture, if the building is 44 meters high?



(36) In the picture, how far apart are the 2 boats if the altitude of the helicopter is 3500 feet?



(37) Consider an isosceles triangle with equal sides of 70 cm each and a base of 80 cm. Find the 3 angles of the triangle.

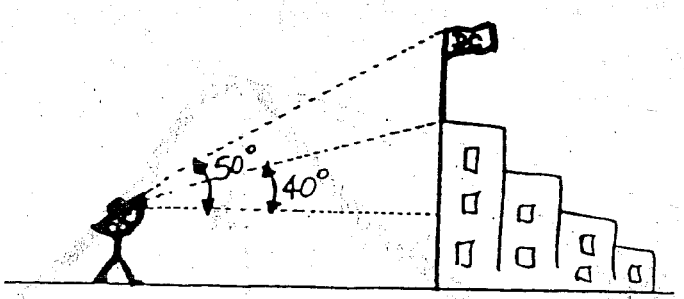
(38) Consider an isosceles triangle where the equal angles are each 50°. If the base is 500 cm, then find the length of the equal sides.

*(39) A great pyramid has a square base 230 m on each side. The 4 triangular faces each make an angle of 51°50' with the ground.

(a) How tall is the pyramid?

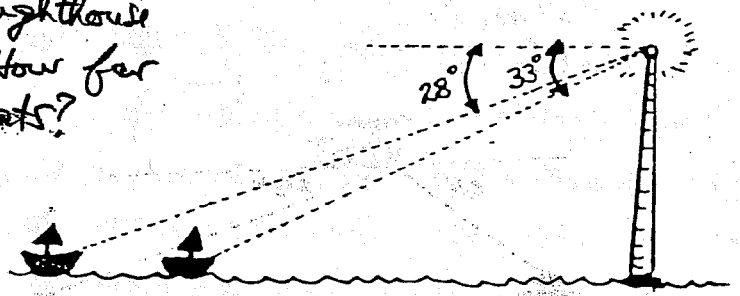
(b) What are the dimensions of each of the triangular faces?

(40) In the picture the surveyor is 6 feet tall and 60 feet from the building. What is the height of the flagpole and the building?



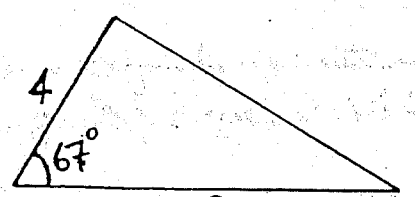
SOLVING RIGHT TRIANGLES - EXERCISES

(41) In the picture the lighthouse is 75 meters high. How far apart are the 2 boats?

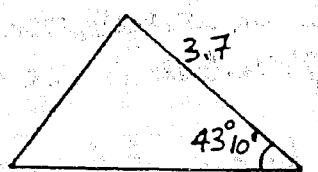


(42) Find the area of each of the following triangles: $(A = \frac{1}{2}bh)$

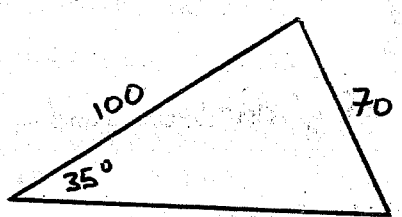
(a)



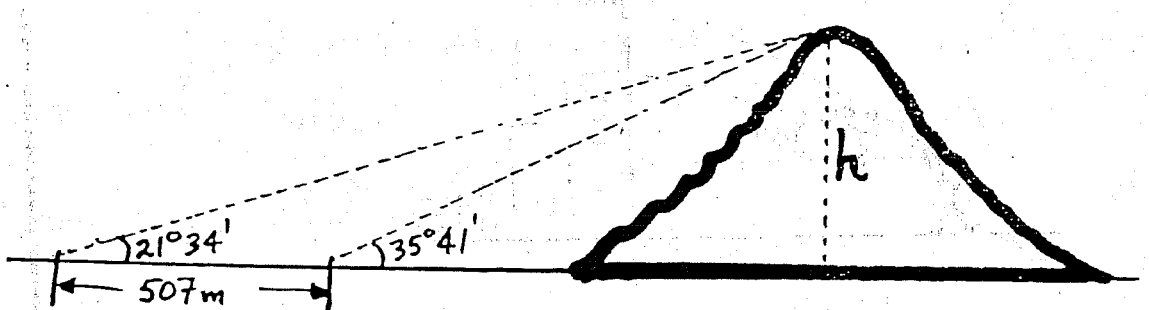
(b)



(c)



(43) In the picture find the height, h, of the mountain.



SOLVING RIGHT TRIANGLES - EXERCISES - ANSWERS

(1) (a) $\sin A = \cos B = \frac{3}{5}$, $\cos A = \sin B = \frac{4}{5}$, $\tan A = \frac{3}{4}$, $\tan B = \frac{4}{3}$

(b) $\sin A = \cos B = \frac{7}{25}$, $\cos A = \sin B = \frac{24}{25}$, $\tan A = \frac{7}{24}$, $\tan B = \frac{24}{7}$

(c) $\sin A = \cos B = \frac{15}{17}$, $\cos A = \sin B = \frac{8}{17}$, $\tan A = \frac{15}{8}$, $\tan B = \frac{8}{15}$

(d) $\sin A = \cos B = 0.5882$, $\cos A = \sin B = 0.8087$, $\tan A = 0.7273$, $\tan B = 1.3750$

(e) $\sin A = \cos B = 0.8212$, $\cos A = \sin B = 0.5698$, $\tan A = 1.4423$, $\tan B = 0.6933$

(f) $\sin A = \cos B = 0.7564$, $\cos A = \sin B = 0.6541$, $\tan A = 1.1564$, $\tan B = 0.8647$

(2) (a) 0.9903 (b) 0.8572 (c) 1.6643 (d) 0.4540 (e) 0.4067 (f) 0.9325

(3) (a) 53.13° (b) 78.46° (c) 36.87° (d) 55.85° (e) 25.58° (f) 53.29°

(4) (a) $B = 17°$, $a = 13.08$, $c = 13.68$ (b) $B = 61°$, $b = 10.82$, $c = 12.38$

(c) $B = 20°$, $b = 36.40$, $c = 106.42$ (d) $B = 35°$, $a = 16.38$, $b = 11.47$

(e) $A = 70°$, $a = 37.59$, $b = 13.69$ (f) $A = 41°$, $b = 3.45$, $c = 4.57$

(g) $A = 38°$, $a = 9.19$, $b = 11.76$ (h) $A = 32°$, $a = 4.37$, $c = 8.25$

(5) (a) $c = 10.63$, $A = 41.19°$, $B = 48.81°$ (b) $a = 9$, $A = 36.87°$, $B = 53.13°$

(c) $c = 41.44$, $A = 19.75°$, $B = 70.25°$ (d) $b = 7$, $A = 73.74°$, $B = 16.26°$

(e) $c = 28.16$, $A = 16.50°$, $B = 73.50°$ (f) $c = 73.57$, $A = 31.10°$, $B = 58.90°$

(g) $a = 47.33$, $A = 61.22°$, $B = 28.78°$ (h) $c = 663.36$, $A = 59.07°$, $B = 30.93°$

(6) (a) 11.49 (b) 350.86 (c) 149.40 (d) 10.7 (e) 79.83 (f) 8.83

(7) 5.97m (8) 1.39m (9) 14.48° (10) 75.52° (11) 11.47ft. (12) 28.53° (13) 44.20m

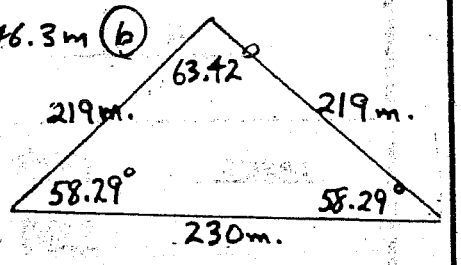
(14) 31.91° (15) 1094ft. (16) 6.70m (17) 18.09m (18) 30.97° (19) 11.09m (20) 553m

(21) (a) 3.18km (b) 80.17° (22) 40m (23) 178ft. (24) 45.68m (25) 284ft.

(26) 18800ft. (27) 2.62km (28) 19.64m (29) 26.79m and 84.53m

(30) 68.6ft. (31) 47.7m (32) 47.7m (33) 14.7m (34) 104.7m (35) 30.15m

(36) 11677ft. (37) 55°, 55° and 70° (38) 389cm (39) (a) 146.3m (b)



(40) 21.2ft. and 51.4ft

(41) 25.6m

(42) (a) $14.7m^2$ (b) $6.1m^2$ (c) $2795.7m^2$

(43) 446m