LAST NAME: SOLUTIONS

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STUDENT NUMBER: ____________

QUIZ 3 (A)

DAWSON COLLEGE

201-NYC-05 Linear Algebra

Instructor: E. Richer

Date: July 10th 2008

Question 1. *(10 marks)*

Let $\vec{u} = (0,2,-1)$, $\vec{v} = (1,-1,-1)$ and $\vec{w} = (1,2,1)$.

Compute the following:

(i) $||2\vec{v}||$

(ii) $\text{proj}_{\vec{v}}\vec{u}$

(iii) $(\vec{u} \times \vec{v}) \times \vec{w}$

(iv) $(3\vec{u}) \cdot \vec{w}$

(v) The volume of the parallelepiped defined by the vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$

(i) $\|2\vec{v}\| = 2\sqrt{1^2+(-1)^2+(-1)^2} = \boxed{2\sqrt{3}}$

(ii) $\text{proj}_{\vec{v}}\vec{u} = \left(\frac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\right)\vec{v} = \frac{-1}{3}(1,-1,-1) = \boxed{\left(\frac{-1}{3}, \frac{1}{3}, \frac{1}{3}\right)}$

(iii) $\underset{\begin{pmatrix}0\\2\\-1\end{pmatrix}\;\begin{pmatrix}1\\-1\\-1\end{pmatrix}}{(\vec{u} \times \vec{v})} \times \vec{w} = \begin{pmatrix}-3\\-1\\-2\end{pmatrix} \times \begin{pmatrix}1\\2\\1\end{pmatrix} = \boxed{(3, 1, -5)}$

(iv) $(3\vec{u})\cdot\vec{w} = (0,6,-3)\cdot(1,2,1) = 0+12-3 = \boxed{9}$

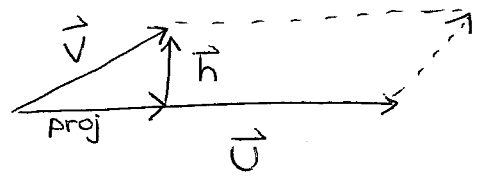
(v) $\vec{u}\cdot(\vec{v}\times\vec{w})$ or $\left|\det\begin{bmatrix}0 & 2 & -1\\ 1 & -1 & -1\\ 1 & 2 & 1\end{bmatrix}\right|$

$= \left|-2\det\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix} - \det\begin{bmatrix}1 & -1\\ 1 & 2\end{bmatrix}\right| = |-2(2)-(3)|$

$= |-7| = \boxed{7}$

Question 2. *(10 marks)*
A parallelogram is defined by the vectors $\vec{v} = (1,1,1)$ and $\vec{u} = (-1,0,2)$.
Compute the **area of the parallelogram** in two different ways:
(i) Using projections
(ii) Using the cross product

(i)



$$\text{proj}_{\vec{u}}\vec{v} = \left(\frac{\vec{u}\cdot\vec{v}}{\vec{u}\cdot\vec{u}}\right)\vec{u} = \frac{1}{5}(-1,0,2) = (-1/5, 0, 2/5)$$

$$\vec{h} = \vec{v} - \text{proj}_{\vec{u}}\vec{v}$$
$$= (1,1,1) - (-1/5, 0, 2/5) = (6/5, 1, 3/5)$$

$$\|\vec{h}\| = \frac{1}{5}\sqrt{6^2+5^2+3^2} = \frac{1}{5}\sqrt{70}$$

$$\|\vec{u}\| = \sqrt{(-1)^2+2^2} = \sqrt{5}$$

$$\text{Area} = \text{base} \times \text{height}$$
$$= \|\vec{u}\| \times \|\vec{h}\|$$
$$= \frac{\sqrt{70}}{5}\sqrt{5}$$
$$= \sqrt{70/5} = \boxed{\sqrt{14}}$$

(ii)

$$\text{Area} = \|\vec{u}\times\vec{v}\| = \left\|\begin{pmatrix}-1\\0\\2\end{pmatrix}\times\begin{pmatrix}1\\1\\1\end{pmatrix}\right\|$$
$$= \|(-2, 3, -1)\|$$
$$= \sqrt{(-2)^2+3^2+(-1)^2} = \boxed{\sqrt{14}}$$