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QUIZ 3 (B)

DAWSON COLLEGE

201-NYC-05 Linear Algebra

Instructor: E. Richer

Date: July 10th 2008

Question 1. *(10 marks)*

Let $\vec{u} = (0,2,-1)$, $\vec{v} = (1,-1,-1)$ and $\vec{w} = (1,2,1)$.

Compute the following:

(i) $||2\vec{w}||$

(ii) $\text{proj}_{\vec{u}}\vec{v}$

(iii) $(\vec{u} \times \vec{w}) \times \vec{v}$

(iv) $(2\vec{u}) \cdot \vec{v}$

(v) The volume of the parallelepiped defined by the vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$

(i) $\|2\vec{w}\| = 2\sqrt{1^2+2^2+1^2} = \boxed{2\sqrt{6}}$

(ii) $\text{proj}_{\vec{u}}\vec{v} = \left(\frac{\vec{u}\cdot\vec{v}}{\vec{u}\cdot\vec{u}}\right)\vec{u} = \frac{-1}{5}(0,2,-1) = \boxed{(0,-2/5,1/5)}$

(iii) $(\vec{u}\times\vec{w})\times\vec{v} = \begin{pmatrix}0\\2\\-1\end{pmatrix}\times\begin{pmatrix}1\\2\\1\end{pmatrix} = \begin{pmatrix}4\\-1\\-2\end{pmatrix}\times\begin{pmatrix}1\\-1\\-1\end{pmatrix} = \boxed{(-1,2,-3)}$

(iv) $2\vec{u}\cdot\vec{v} = (0,4,-2)\cdot(1,-1,-1) = \boxed{-2}$

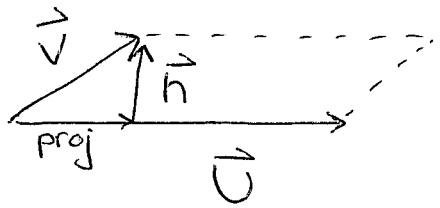
(v) $|\vec{u}\cdot(\vec{v}\times\vec{w})|$ or

$$\left|\det\begin{bmatrix}0&2&-1\\1&-1&-1\\1&2&1\end{bmatrix}\right| = \left|-2\det\begin{bmatrix}1&-1\\1&1\end{bmatrix} - \det\begin{bmatrix}1&-1\\1&2\end{bmatrix}\right|$$

$$= |-2(2) - 3| = |-7| = \boxed{7}$$

Question 2. *(10 marks)*
A parallelogram is defined by the vectors $\vec{v} = (1,-1,1)$ and $\vec{u} = (-1,0,2)$.
Compute the **area of the parallelogram** in two different ways:
(i) Using projections
(ii) Using the cross product

(i)



$$\text{proj}_{\vec{u}}\vec{v} = \left(\frac{\vec{v}\cdot\vec{u}}{\vec{u}\cdot\vec{u}}\right)\vec{u}$$

$$= \frac{1}{5}(-1,0,2) = \left(-\tfrac{1}{5}, 0, \tfrac{2}{5}\right)$$

$$\vec{h} = \vec{v} - \text{proj}_{\vec{u}}\vec{v}$$

$$= (1,-1,1) - \left(-\tfrac{1}{5}, 0, \tfrac{2}{5}\right) = \left(\tfrac{6}{5}, -1, \tfrac{3}{5}\right)$$

$$\text{Area} = \|\vec{h}\|\|\vec{u}\|$$

$$= \frac{1}{5}\sqrt{6^2+(-5)^2+3^2}\ \sqrt{(-1)^2+2^2}$$

$$= \frac{1}{5}\sqrt{70}\ \sqrt{5} = \boxed{\sqrt{14}}$$

(ii)

$$\|\vec{u}\times\vec{v}\| = \left\| \begin{pmatrix}-1\\0\\2\end{pmatrix} \begin{pmatrix}1\\-1\\1\end{pmatrix} \right\| = \|(2,3,1)\|$$

$$= \sqrt{2^2+3^2+1^2}$$

$$= \boxed{\sqrt{14}}$$